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Abstract

This dissertation consists of three chapters that analyze international trade and international lending.

Chapter 1 studies the link between trade and the correlation of business cycle fluctuations across countries. The data show a positive relationship between trade intensity and the correlation of Gross Domestic Product (GDP). The standard international real business cycle model is extended to incorporate two features of trade theory: endogenous specialization and trade in intermediate goods. Countries with closer trade relationships therefore trade a larger set of goods, and also rely on each other more intensively for inputs to production; both effects generate closer comovement of GDP.

Chapter 2 develops a model of trade in intermediate inputs with heterogeneous producers to analyze the dynamics of aggregate trade flows in response to movements in the relative price of imported to domestic goods. In aggregate data, trade volumes adjust slowly in response to relative price changes, a prediction at odds with standard theories. The main feature of the model is the producer-level irreversibility in the decision to use imported inputs. When calibrated to match cross-section data on producer heterogeneity in the use of imported intermediates, the model here generates a slow response of the volume of trade in response to relative price changes.

Chapter 3 builds a dynamic model of international lending and default to study the optimal maturity composition of sovereign debt and the term structure of emerging market interest rates. In emerging markets data, long maturity bonds are issued mostly in tranquil times even though the interest rate spreads are higher than for short-maturity bonds. In crises times, short maturity debt is issued and the interest rate spreads are decreasing in maturity. The model generates the observed movements in interest rate spread curves through the timing of default risk. The model also predicts that long debt is issued primarily in tranquil times because it provides insurance against future bad shocks.

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Introduction

Since the middle of the twentieth century, the world economy has become increasingly integrated. Worldwide international merchandise trade has grown faster than the world's industrial output, so that countries trade more and more of what they produce. At the same time, trade in financial assets has grown, especially among the emerging market nations of Latin America, Eastern Europe, and Asia. Economists and policy-makers are increasingly interested in the effects of international trade and financial flows on a country's economy. The three chapters of this dissertation address three questions related to the dynamic implications of openness to international trade and international financial flows. Roughly, these questions are: Why do countries with closer trade relationships have more correlated business cycles? Why does the volume of goods a country imports respond slowly to changes in the price of imports? Finally, why do emerging market governments face such high and volatile interest rates for borrowing from international lenders, and why does the pattern of these rates across debt maturities vary over time?

Chapter 1 studies the link between trade intensity and the correlation of business cycle fluctuations across countries. The data show a positive relationship between trade intensity and the correlation of Gross Domestic Product (GDP). The standard international real business cycle model is extended to incorporate two features of trade theory: endogenous specialization and trade in intermediate goods. Countries with closer trade relationships therefore trade a larger set of goods, and also rely on each other more intensively for inputs to production; both effects generate closer comovement of GDP. The model can qualitatively match the positive effect of trade on GDP correlations, but does not quantitatively capture the magnitude of this effect in the data.

Chapter 2 develops a model of trade in intermediate inputs with heterogeneous producers to analyze the dynamics of aggregate trade flows in response to movements in the relative price of imported to domestic goods. In aggregate data, trade volumes adjust slowly in response to relative price changes, a prediction at odds with standard theories. The main feature of the model is the producer-level irreversibility in the decision to use imported inputs. When calibrated to match cross-section data on producer heterogeneity in the use of imported intermediates, the model here generates a slow response of the volume of trade in response to relative price changes. Relative price movements induce immediate changes in aggregate imported relative to domestic purchases through adjustment within importing producers, and through the reallocation of resources between non-importing and importing producers. Additionally, trade volumes adjust slowly through gradual changes in the fraction of importers in the economy. This slow adjustment in aggregate trade flows significantly affects the measurement of the welfare gains from trade policy reform: the slow growth of trade following a reform reduces estimated welfare gains.

Chapter 3 builds a dynamic model of international lending and default to study the optimal maturity composition of sovereign debt and the term structure of emerging market interest rates. In emerging markets data, long maturity bonds are issued mostly in tranquil times even though the interest rate spreads are higher than for short-maturity bonds. In crises times, short maturity debt is issued and the interest rate spreads are decreasing in maturity. The model generates these observed movements in the interest rate spread curves through the endogenous probability of default. The spread curve is upward sloping in tranquil times because only the long spread will reflect the likelihood of a default far in the future. However, if a default is likely in the near future the spread curve is inverted because the economy may repay its debt obligations in all future states if it avoids the stressed period. When calibrated to data from Brazil, the model matches various features of the data, including the dynamics of the spread curve and the volatility of short- and long-maturity bond spreads. The model also predicts that long debt is issued primarily in tranquil times because it provides insurance against future bad shocks.

Chapter 1

Trade Intensity and International Comovement with Endogenous Specialization and Intermediate Goods

1.1 Introduction

Recent empirical studies find that increased trade induces closer comovement of output fluctuations between trading partners. Frankel and Rose (1998) find in cross-section data for many countries that tighter trade relationships are associated with higher business cycle comovement.[1] Kose and Yi (2001, 2006) have illustrated a trade-comovement puzzle: the standard international real business cycle model of Backus et al. (1994) cannot quantitatively account for the relationship between trade and comovement of GDP.

In this paper, we construct and quantitatively assess a model that has the potential

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[1] Other recent studies with similar results include Baxter and Kouparitsas (2004) and Kose and Yi (2002).

to generate such a relationship between trade and business cycle comovement. The model builds on a two-country model of international business cycles driven by productivity shocks, as in Backus et al. (1992 and 1994). We add two features to the production and trade structure of the model motivated by international trade theory: endogenous specialization in the set of goods each country produces and trade in intermediate inputs to production.

Endogenous specialization is modeled as in Dornbusch et al. (1977) and departs from the fixed patterns of specialization based on Armington (1969), an assumption that is embodied in the standard international business cycle literature. The feature of endogenous specialization allows for the transmission of the shocks between countries that trade more intensively through trade over a larger set of goods. Trade in intermediate goods, modeled as in Eaton and Kortum (2002), has the potential to transmit aggregate shocks between countries as foreign inputs are necessary for domestic production.

In our numerical simulations, the model can qualitatively generate the positive effect of trade on GDP correlations in an artificially generated cross-section of model economies. However, it does not quantitatively capture the relationship between trade intensity and GDP correlation. To understand this shortcoming, we present data that indicate a positive relationship between trade intensity and the correlation of Total Factor Productivity (TFP). We find that the inability of the model to quantitatively match the relationship between trade and correlation of GDP is mainly due to the fact that the model does not imply any effect of trade on the correlation of TFP as measured in the data.

Models of international business cycles have largely relied on trade patterns generated by national differentiation of commodities, as in Armington (1969). The process of aggregating imports and domestically produced goods–the so-called "Armington aggregator"–assumes that goods from different countries are intrinsically imperfect substitutes, combined according to a constant elasticity of substitution. Recently, however, several papers have incorporated more sophisticated theories of trade to quantitatively examine movements of macroeconomic variables.[2] Kose and Yi (2001) suggest that models based on the Armington aggregator cannot explain the dependence of business cycle correlations on trade because specialization patterns do not depend on trade intensity; for example, specialization patterns do not respond to changes in trade policy. In addition, we consider the feature of intermediate goods to be important in generating coordination of demands by the producing industries in each country. By modeling intermediate inputs as in Eaton and Kortum (2002), and assuming endogenous specialization patterns, producers of final output require inputs from each country.[3] A closer trade relationship implies that a higher volume as well as a larger set of foreign intermediate goods are used for the domestic production.

The rest of the paper is organized as follows: Section 2 briefly covers some facts in the data studied by the papers mentioned in the opening paragraph, as well as a few statistics to provide some direct evidence on the mechanisms we suggest. Section 3 presents the model, and Section 4 displays the model's quantitative implications for the effects of trade on comovement.

1.2 Data

This section aims to provide some numbers for comparison with our model's results. The data analysis here is far from exhaustive, though it would be worthwhile to further study the statistics presented here, for example, for a broader set of countries. We present data on the dependence of cross-country correlations of GDP and TFP on trade, and simple statistics on the volume of intermediate goods trade and the differences in specialization patterns among trading partners.

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[2] See for example, Melitz and Ghironi (2004) or Alessandria and Choi (2004) or more closely related to the Ricardian framework of our paper, Yi (2003).

[3] Burstein et al. (2004) study the dependence of GDP correlations on trade within an Armington aggregator-based model with intermediate inputs.

1.2.1 Cross-section cross-correlation regressions

We follow Frankel and Rose (1998), as well as others previously mentioned in estimating the following relationship between one of the correlations in which we are interested (GDP or TFP) and trade:

$$Corr_{ij} = \beta_0 + \beta_1 \log(Trade_{ij}) + \varepsilon_{ij} \ . \tag{1.1}$$

This is a cross-section regression, where ij denotes a variable associated with the bilateral relationship between country i and country j. $Corr_{ij}$ is the correlation of logged and Hodrick-Prescott-filtered (HP-filtered) annual real GDP (or TFP).[4] $Trade_{ij}$ is the ratio of total trade between the two countries (measured as the sum of each country's imports from the other) to total GDP:

$$Trade_{ij} = \frac{imports_{ij} + imports_{ji}}{GDP_i + GDP_j} \ ,$$

where $imports_{ij}$ denotes imports by country i from country j, and GDP_i denotes GDP in country i at current prices.

Table 1.1 reports the coefficients from OLS estimation of (1.1), using GDP and TFP correlations as dependent variables in two different regressions. The countries are included are detailed in the Appendix. The interpretation of the value of the coefficient β_1 is that a doubling of trade intensity $Trade_{ij}$ between a pair of countries results in an increase of $\beta_1 \times \log(2)$ in the correlation of interest. For GDP, this increase would be 0.065, and for the TFP correlation, this increase would be 0.038. The constants give the expected correlation of a pair of trading partners for whom $Trade_{ij}$ would be 100%. Our numbers are similar to those of Kose and Yi (2006).

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[4] TFP is constructed as the usual "Solow residual" from a Cobb-Douglas aggregate production function, $TFP = GDP/\left(K^{\alpha}L^{1-\alpha}\right)$, K denoting capital and L denoting labor. See Appendix 1 for more description.

1.2.2 Trade Specialization Patterns

Empirical studies using detailed goods data, have revealed that specialization patterns of trade substantially differ across different countries and also for a given country but across different time periods. In particular, Hummels and Klenow (2002) have reported a large variation in the sets of goods traded across different trading partners. In particular, higher trade volumes typically translate to both higher trade per good but also to a higher number of traded goods. On the other hand Kehoe and Ruhl (2002) study the trade relationship of given pairs of countries but in different time periods following an event of a trade liberalization. They find that a large part of the increase in trade after the liberalization is due to goods that had little trade before the liberalization. They interpret this as evidence of trade in new goods. Consistent with both findings, in our model countries that trade more trade also a larger set of goods.

1.2.3 Intermediate Goods

The role of intermediate goods in trade has been studied previously by Hummels et al. (1998) and Hummels et al. (2001). These papers define particular statistics for measuring the extent of trade in intermediate goods. In Hummels et al. (2001), for example, the measured statistic corresponds to the percentage of the value of exported goods that is attributed to imported intermediate inputs. For the United States, this number grows from 6% of total exports to all partners in 1970 to 11% in 1990. In Mexico, the corresponding number was between 5 and 10% in the early 1980's and grew to 30% in the late 1990's. Hummels et al. (1998) provide data suggesting that a similar statistic for the particular bilateral trade relationship between the US and Mexico grows from 20% in 1975 to nearly 40% in 1995.

1.3 Model

The model is a two-country real business cycle model in the tradition of Backus et. al (1992, 1994), modified to include a continuum of tradeable goods. The time horizon is infinite and discrete, and periods are indexed by $t = 0, 1, \ldots$. Countries are indexed by $i, j = 1, 2$, and goods are indexed by $z, \zeta \in [0, 1]$. Subscripts refer to time periods and superscripts refer to countries.

1.3.1 Households

Each country i is populated by an infinitely-lived representative household who values sequences of consumption of every good $z \in [0, 1]$, consumption of a non-tradable good, and leisure, according to the following preferences:[5]

$$E \sum_{t=0}^{\infty} \beta^t \left(\left(\left(\int_0^1 c_t^i(z)^\rho dz \right)^{\gamma/\rho} \left(C_{Nt}^i \right)^{1-\gamma} \right)^{\mu} \left(1 - L_t^i \right)^{1-\mu} \right)^{1-\sigma} \Big/ (1 - \sigma) \,, \tag{1.2}$$

where L_t^i denotes the fraction of time devoted to labor services supplied to domestic industries. E denotes the expectation over the entire time horizon, and $\beta \in (0, 1)$ is the household's discount factor. The household receives income from selling labor services and renting capital to firms in each period, along with lump-sum transfers of tariff revenue (if any), and purchases consumption and investment goods. The budget constraint of the household in country i is:

$$\int_0^1 p_t^i(z) \left(c_t^i(z) + x_t^i(z) \right) dz + P_{Nt}^i \left(C_{Nt}^i + X_{Nt}^i \right) \leq w_t^i L_t^i + r_t^i K_t^i + T_t^i \,. \tag{1.3}$$

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[5] Under free trade, countries will be completely specialized in production of tradeable goods. If the model contained only these goods, trade volumes would be implausibly high. One way to deal with this would be to incorporate transport costs in addition to tariffs. We choose to instead model an exogenously non-tradeble sector because tariffs and transport costs affect the pattern of trade in the same way, and we would like to examine the implications of these effects for a broad range of tariffs. Imposing high transport costs to induce a "home bias" in consumption would limit the range in which we can vary tariffs and still have positive trade in equilibrium.

This formulation implies an absence of international trade in financial assets between households. That is, trade in goods is balanced in each period.

The country's capital stock is accumulated by the household according to:

$$K_{t+1}^i = \left(\int_0^1 x_t^i(z)^\rho dz \right)^{\gamma/\rho} \left(X_{Nt}^i \right)^{1-\gamma} + (1-\delta) K_t^i \,, \tag{1.4}$$

where $\delta \in (0,1)$ is the depreciation rate of capital.

Note that we have assumed that households bundle goods from each tradeable industry and the nontradeable sector together to form new investment in the same way that they bundle goods to form utility from consumption. We may easily modify the model to allow for differences in the bundling of consumption and investment, but the case in which they are the same seems a plausible starting point.

1.3.2 Production

Production of a continuum of tradeable goods in both countries is modeled as in Dornbusch, Fisher and Samuelson (1977), adapted to include physical capital and intermediate goods. The production structure of tradeable goods is essentially a simplification of the "vertical specialization" model in Yi (2003). Any good $z \in [0,1]$ can be produced under perfect competition in country $i = 1, 2$ at time t using capital, labor, and a continuum of intermediate goods as inputs, according to

$$y_t^i(z) = \left(A_t^i(z) k_t^i(z)^\alpha \ell_t^i(z)^{1-\alpha} \right)^\theta \left(\int_0^1 m_t^i(z,\zeta)^\rho d\zeta \right)^{(1-\theta)/\rho} \,, \tag{1.5}$$

where $y_t^i(z), k_t^i(z)$, and $\ell_t^i(z)$ denote output, capital and labor, respectively, in industry z, and $m_t^i(z,\zeta)$ denotes the quantity of intermediate good of type ζ used by industry z. Intermediate goods themselves are produced using the production function above, so that any of the tradeable goods is both an intermediate and a final good. This formulation has been used, for example, by Eaton and Kortum (2002) and allows for the observation in disaggregated input-output tables that most industries use goods from all other industries as inputs. The words "good" and "industry" are interchangeable here, as they essentially are in input-output tables.

$A_t^i(z)$ denotes country- and industry-specific total factor productivity. We assume that it takes the form of a stochastically time-varying aggregate component and a constant industry component:

$$A_t^i(z) = A_t^i \times \varphi^i(z) \ .$$

Output of each good z is allocated towards consumption, investment, and intermediate usage by all industries in both countries, according to:

$$c_t^1(z) + c_t^2(z) + x_t^1(z) + x_t^2(z) + \int_0^1 m_t^1(\zeta, z) d\zeta + \int_0^1 m_t^2(\zeta, z) d\zeta = y_t^1(z) + y_t^2(z) \ . \quad (1.6)$$

When we describe patterns of trade, we will see that countries are completely specialized for certain ranges of goods, and incompletely specialized in other ranges, due to the presence of nontraded goods arising from tariffs. That is, for traded goods, either $y_t^1(z)$ or $y_t^2(z)$ will be zero, and for nontraded goods,

$$c_t^i(z) + x_t^i(z) + \int_0^1 m_t^i(\zeta, z) d\zeta = y_t^i(z) \ ,$$

for both $i = 1, 2$.

The bundling of intermediate goods used in production is already evident in the production function (1.5), but we will redefine for clarity:

$$M_t^i(z) = \left(\int_0^1 m_t^i(z, \zeta)^\rho d\zeta \right)^{1/\rho} \ , \quad (1.7)$$

as a composite intermediate good used by industry z.

The non-tradeable good in each country is produced using the technology:

$$Y^i_{Nt} = A^i_t \left(K^i_{Nt}\right)^{\alpha} \left(L^i_{Nt}\right)^{1-\alpha} ,$$

and is allocated to consumption and investment according to:

$$\begin{aligned} C^1_{Nt} + X^1_{Nt} &= Y^1_{Nt} , \\ C^2_{Nt} + X^2_{Nt} &= Y^2_{Nt} . \end{aligned} \tag{1.8}$$

1.3.3 Equilibrium

We consider a competitive equilibrium with free entry and exit in every industry, in which imports of all intermediate and final goods in each country are subject to a uniform tariff at the rate τ.

An equilibrium, given the tariff rate τ, consists of stochastic processes for each country $i = 1, 2$ for factor prices, w^i_t, r^i_t; producer prices for tradeable goods, $q^i_t(z)$; purchaser prices for tradeable goods, $p^i_t(z)$; prices for nontradeable goods P^i_{Nt}; an allocation of tradeable goods, $c^i_t(z), x^i_t(z), m^i_t(z,\zeta), y^i_t(z), k^i_t(z), \ell^i_t(z)$; an allocation of nontradeable goods, $C^i_{Nt}, X^i_{Nt}, Y^i_{Nt}, K^i_{Nt}, L^i_{Nt}$; aggregate factors, K^i_t, L^i_t; and tariff rebates, T^i_t such that:

1. Given the prices $w^i_t, r^i_t, p^i_t(z), P^i_{Nt}$, the functions $c^i_t(z), x^i_t(z)$ and quantities $C^i_{Nt}, X^i_{Nt}, K^i_t, L^i_t$ solve the problem of maximizing (1.2) subject to (1.3) and (1.4) for each country $i = 1, 2$.

2. The prices $w^i_t, r^i_t, q^i_t(z), p^i_t(z)$, inputs $m^i_t(z,\zeta), k^i_t(z), \ell^i_t(z)$ and output levels $y^i_t(z)$ satisfy:

   a) $q^i_t(z)\theta\alpha \left(A^i_t(z)k^i_t(z)^{\alpha}\ell^i_t(z)^{1-\alpha}\right)^{\theta-1} A^i_t(z)k^i_t(z)^{\alpha-1}\ell^i_t(z)^{1-\alpha}M^i_t(z)^{1-\theta} \leq r^i_t,$

   with equality if $y^i_t(z) > 0$.

   b) $q_t^i(z)\theta(1-\alpha)\left(A_t^i(z)k_t^i(z)^{\alpha}\ell_t^i(z)^{1-\alpha}\right)^{\theta-1} A_t^i(z)k_t^i(z)^{\alpha}\ell_t^i(z)^{-\alpha}M_t^i(z)^{1-\theta} \leq w_t^i,$

   with equality if $y_t^i(z) > 0$.

   c) $q_t^i(z)(1-\theta)\left(A_t^i(z)k_t^i(z)^{\alpha}\ell_t^i(z)^{1-\alpha}\right)^{\theta}\left(\int_0^1 m_t^i(z,\zeta)^{\rho}d\zeta\right)^{(1-\theta)/\rho-1} m_t^i(z,\zeta)^{\rho-1} \leq p_t^i(\zeta),$

   with equality if $y_t^i(z) > 0$.

   where $M_t^i(z)$ is defined in (1.7).

3. The prices w_t^i, r_t^i, P_{Nt}^i, inputs K_{Nt}^i, L_{Nt}^i, and output levels Y_{Nt}^i satisfy:

   a) $P_{Nt}^i \alpha A_t^i \left(K_{Nt}^i\right)^{\alpha-1}\left(L_{Nt}^i\right)^{1-\alpha} = r_t^i$

   b) $P_{Nt}^i(1-\alpha)A_t^i\left(K_{Nt}^i\right)^{\alpha}\left(L_{Nt}^i\right)^{-\alpha} = w_t^i$

4. The functions $c_t^i(z), x_t^i(z), m_t^i(z,\zeta), y_t^i(z)$ satisfy (1.6).

5. C_{Nt}^i, X_{Nt}^i and Y_{Nt}^i satisfy (1.8).

6. $K_t^i = \int_0^1 k_t^i(z)dz + K_{Nt}^i$

   $L_t^i = \int_0^1 \ell_t^i(z)dz + L_{Nt}^i$

7. $p_t^i(z) = \min\left\{q_t^i(z), (1+\tau)q_t^j(z)\right\}$ for $j \neq i$

8. $T_t^i = \int_{\{\zeta: q_t^i(\zeta) > (1+\tau)q_t^j(\zeta)\}} \tau q_t^j(\zeta)\left(c_t^i(\zeta) + x_t^i(\zeta) + \int_0^1 m_t^i(z,\zeta)dz\right)d\zeta$

Items 2 and 3 incorporate optimization by firms in each tradeable industry and the nontradeable sector. Items 4 and 5 impose physical feasibility for each good. Item 6 equates factor supplies by households in each country with factor demands from domestic industries. Item 7 defines consumer prices and determines the pattern of trade. Item 8 defines tariff rebates as revenue collected from tariffs on imports in each country. Items 7 and 8 embody the fact that specialization patterns arise as the result of purchasers of tradeable goods choosing between countries for the lowest price for each good, considering that imports would be subject to the tariff.

1.3.4 Pattern of Trade and Equilibrium Computation

In computing an equilibrium numerically, we make use of the implied pattern of trade, and of auxiliary optimization problems framed in terms of composite consumption, investment and intermediate goods. The objective is to write a system of difference equations in terms of aggregate variables, which can then be solved using linear approximation methods.[6] For example, we imagine the household in each country choosing aggregate tradeable consumption and investment (C_t^i and X_t^i) and nontradeable consumption and investment (C_{Nt}^i and X_{Nt}^i), along with labor L_t^i and capital stock K_t^i in order to solve:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left(\left((C_t^i)^\gamma \left(C_{Nt}^i\right)^{1-\gamma} \right)^\mu (1 - L_t^i)^{1-\mu} \right)^{1-\sigma} \Big/ (1-\sigma)$$

subject to:

$$P_t^i \left(C_t^i + X_t^i\right) + P_{Nt}^i \left(C_{Nt}^i + X_{Nt}^i\right) \le w_t^i L_t^i + r_t^i K_t^i + T_t^i$$

$$K_{t+1}^i = \left(X_t^i\right)^\gamma \left(X_{Nt}^i\right)^\gamma + (1-\delta) K_t^i \ .$$

Meanwhile, an intermediary firm in each country purchases quantities of each tradeable good, $c_t^i(z)$ at prices $p_t^i(z)$ in order to form composite consumption to sell to the household at price P_t^i,

$$\max P_t^i C_t^i - \int_0^1 p_t^i(z) c_t^i(z) dz \tag{1.9}$$

subject to:

$$C_t^i = \left(\int_0^1 c_t^i(z)^\rho dz \right)^{1/\rho} \ .$$

and similarly for composite tradeable investment X_t^i and intermediates used in each industry z, $M_t^i(z)$.

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[6] Similar approaches have been used by Naknoi (2004) and Bergin and Glick (2004), in models with a continuum of goods.

The optimization in (1.9) yields demands $c_t^i(z)$ as a function of $C_t^i, p_t^i(z)$, and P_t^i:

$$c_t^i(z) = C_t^i \left(\frac{p_t^i(z)}{P_t^i} \right)^{1/(\rho-1)} ,$$

and gives P_t^i in turn as a function of $p_t^i(z)$ for all goods z:

$$P_t^i = \left(\int_0^1 p_t^i(z)^{\rho/(\rho-1)} \right)^{(\rho-1)/\rho} .$$

In the computations, we designate the composite tradeable good in country 1 as the numeraire (so $P_t^1 = 1$ for all t), so that prices of all goods and factors are in units of composite tradeable consumption in country 1.

The pattern of trade is determined in equilibrium: each country will import a good or produce it domestically according to which option would result in a lower consumption price, $p_t^i(z)$. From the cost-minimization problems of producers, the cost of producing any good z is given by:

$$q_t^i(z) = \left(\frac{\left(w_t^i\right)^{(1-\alpha)} \left(r_t^i\right)^{\alpha}}{\theta A_t^i(z) \alpha^{\alpha} \left(1-\alpha\right)^{(1-\alpha)}} \right)^{\theta} \left(\frac{\left(P_t^i\right)}{(1-\theta)} \right)^{1-\theta} . \tag{1.10}$$

If producing good z, a firm in country i will sell its output at price $q_t^i(z)$. The price paid by the consumer in country j is $q_t^j(z)$ for domestically produced goods z, and $(1+\tau)q_t^i(z)$ for imported goods. This leads to the following characterization of the trade pattern:

1. For z such that

   $$q_t^1(z) < \frac{1}{1+\tau} q_t^2(z) , \tag{1.11}$$

   firms in country 1 are the sole producers.

2. For z such that

   $$q_t^2(z) < \frac{1}{1+\tau} q_t^1(z) , \tag{1.12}$$

firms in country 2 are the sole producers.

3. For z such that
$$\frac{1}{1+\tau}q_t^2(z) \leq q_t^1(z) \leq (1+\tau)q_t^2(z) \ , \tag{1.13}$$
firms in both countries produce, and the goods are not traded.

As can be seen from examining (1.10) along with the three inequalities above, the determination of each range of goods depends on relative input costs in each country, as measured by $w_t^1/w_t^2, r_t^1/r_t^2$, and P_t^1/P_t^2, as well as on relative productivities $A_t^1\varphi^1(z)/A_t^2\varphi^2(z)$. In what follows, we will assume a simple form for the industry productivity functions:

$$\begin{aligned} \varphi^1(z) &= 1+\lambda(1-z) \ , \\ \varphi^2(z) &= 1+\lambda z \ . \end{aligned} \tag{1.14}$$

For this example, the pattern of trade simply takes the form of two cutoffs, z_t^ℓ and z_t^h, separating the three ranges of goods described above. Choosing functions so that relative productivity, $\varphi^1(z)/\varphi^2(z)$, is decreasing yields this very simple pattern of trade. In general, whatever the form of these functions, the pattern of trade will take the form of a finite number of cutoffs separating sets of goods that fall into one of the three categories above.[7]

As the cutoffs depend on equilibrium prices, we treat the cutoffs as equilibrium variables to be solved for, and each is determined by the boundaries of the sets described in 1. and 2. above.

Given an equilibrium pattern of trade in period t, we determine each industry's output, in each country, from the feasibility condition (1.6), in terms of consumption, investment, and intermediate goods expenditures. For example, since country 1 produces goods $z \in$

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[7] The functions in (1.14) also impose a strong symmetry between countries, in the sense that, for any good z_1 at which country 1 has a certain productivity, there is a corresponding good $z_2 = 1 - z_1$ for which country 2 has the same productivity. Our method for computing equilibrium does not, however, depend on this symmetry.

$[z_t^\ell, z_t^h]$ only for domestic consumption,

$$y_t^1(z) = c_t^1(z) + x_t^1(z) + \int_0^1 m_t^1(\zeta, z)d\zeta$$

for all such z.

Expenditures on different goods are in turn determined in terms of composite consumption, investment, and intermediates, and aggregate and industry prices, from intermediary problems like (1.9),so

$$y_t^1(z) = \left(\frac{p_t^1(z)}{P_t^1}\right)^{1/(\rho-1)} (C_t^1 + X_t^1 + M_t^1)$$

where we have defined $M_t^i = \int_0^1 M_t^i(z)dz$ as total consumption of the composite intermediate good by all industries in country i.

Consumer prices are functions of wage and rental rates, and industry productivities (as in (1.10)): for $z \in [0, z_t^\ell]$, $p_t^1(z) = q_t^1(z)$, so

$$y_t^1(z) = \left(q_t^1 \varphi^1(z)^{-\theta}\right)^{1/(\rho-1)} \left(\frac{1}{P_t^1}\right)^{1/(\rho-1)} \left(C_t^1 + X_t^1 + M_t^1\right) ,$$

where we have separated $q_t^1(z)$ into an aggregate component,

$$q_t^1 = \left(\left(w_t^1/(1-\alpha)\right)^{1-\alpha} \left(r_t^1/\alpha\right)^\alpha \left(A_t^1\right)^{-1}\right)^\theta \left(P_t^1/(1-\theta)\right)^{1-\theta} ,$$

and an industry component, $\varphi^1(z)^{-\theta}$. Relating the above formula to industry factor demands,

$$k_t^1(z) = \frac{\alpha q_t^1(z) y_t^1(z)}{r_t^1} .$$

Solving for the aggregate capital stock is just a matter of integrating over industry demands.

$$\int_{z_t^\ell}^{z_t^h} k_t^1(z)dz = \left(r_t^1\right)^{-1} \alpha \left(q_t^1\right)^{\rho/(\rho-1)} \left(\frac{1}{P_t^1}\right)^{1/(\rho-1)} \left(C_t^1 + X_t^1 + M_t^1\right) \int_{z_t^\ell}^{z_t^h} \varphi^1(z)^{-\theta\rho/(\rho-1)} dz \ . \tag{1.15}$$

The right hand side of (1.15) now only depends on aggregate variables and a given function whose integral is known. Similarly, $k_t^1(z)$ can be determined for the range in which country 1 specializes in production, $[0, z_t^\ell]$. The factor market clearing condition for capital is then

$$K_t^1 = \int_0^{z_t^\ell} k_t^1(z)dz + \int_{z_t^\ell}^{z_t^h} k_t^1(z)dz + K_{Nt}^i \ .$$

We reduce all the equilibrium conditions pertaining to the continuum of tradeable goods in this way to a system of equations in terms of aggregate factors (K, L), composite quantities and aggregate prices (C, X, M, P, w, r), aggregate technology shocks (A), and pattern-of-trade cutoffs (z^ℓ, z^h). (Equilibrium conditions involving the non-tradeable sector are already in terms of aggregate quantities.)

We solve the model numerically by a log-linear approximation of this set of equations around the steady state of the model's deterministic analogue, following the methods in, for example, Klein (1997).

1.3.5 Model Statistics

Our main focus is on the correlation in GDP between two countries. In order to compare the model's predictions to the data, we need to construct a comparable measure of GDP from the model's output. Since our results depend heavily on the method of GDP measurement, we dwell on this point a bit here. In our model with a continuum of goods and changing trade patterns, we have some choice regarding how to compute aggregate quantities. We would like to construct an analogue of the statistic of real GDP, that is, GDP in base period prices, as reported in actual data by national statistical agencies. In order to do this in a way that is as close as possible to the methods used by these agencies, our guidelines for this procedure are the recommendations provided by the UN's *System of National Accounts 1993* (SNA 93).

Our definition of GDP at current prices is the given by aggregate gross output,

$$GO_t^i = \int_{\Omega_t^i} q_t^i(z) y_t^i(z) dz + P_{Nt}^i Y_{Nt}^i \ ,$$

minus aggregate intermediate consumption,

$$IC_t^i = \int_{\Omega_t^i} \left[\int_0^1 p_t^i(\zeta) m_t^i(z,\zeta) d\zeta \right] dz \ ,$$

plus tariffs on imports,

$$T_t^i = \int_{[0,1]/\Omega_t^i} \tau q_t^j(z)\, y_t^i(z)\, dz,$$

$$Y_t^i = GO_t^i - IC_t^i + T_t^i \ . \tag{1.16}$$

In the above formulas, Ω_t^i denotes the subset of $[0,1]$ for which country i is producing at period t. We value output of a good at its producer's price, $q_t^i(z)$, and intermediate consumption of a good at it's purchaser's (consumer's) price, $p_t^i(\zeta)$ (see *SNA 93*, paragraph 6.37). The difference in the prices $q_t^i(z)$ and $p_t^i(z)$ is in the point of measurement; in the model, the only difference between the two is that the latter includes tariffs on imported goods. To be consistent with accounting on the expenditure side, where imports are valued in producers prices, it follows that import tariffs must be added to our definition of GDP (see *SNA 93*, paragraph 6.235)

Y_t^i is a measure of country i's aggregate value added in units of the composite consumption good in country 1 at period t, C_t^1. We would, however, like to consider a measure of GDP at constant prices, as measured in the data. Since the basket of goods produced in each country changes over time, we want to measure GDP in different periods in units of a fixed production basket. So, for our measure of GDP at constant prices, we pick a base period (period 0) and reconstruct the above formulas, using base period prices:

$$\widehat{GO_t^i} = \int_{\Omega_t^i} q_0^i(z) y_t^i(z) dz + P_{N0}^i Y_{Nt}^i$$

$$\widehat{IC_t^i} = \int_{\Omega_t^i} \left[\int_0^1 p_0^i(\zeta) m_t^i(z,\zeta) d\zeta \right] dz,$$

$$\widehat{T}_t^i = \int_{[0,1]/\Omega_t^i} \tau q_0^j \left(z\right) y_t^i \left(z\right) dz,$$

and

$$\hat{Y}_t^i = \widehat{GO_t^i} - \widehat{IC_t^i} + \widehat{T}_t^i \ . \tag{1.17}$$

Effectively, we are using what national statistical agencies refer to as a "double-deflation" method, deflating gross output and intermediate consumption each by their own deflators.

A practical problem that this method raises is that country-specific period-0 producer prices are not well defined for all goods, due to the fact that specialization patterns change in the model. For example, it may be the case that good z is produced in country 1 in period t, but was not produced by country 1 in period 0. In this case, it is unclear at what price we should value country 1's output of good z at period t in calculating real gross output. This is surely a problem in actual national accounting as well, as products are newly invented or disappear through time, and must be assessed base period prices in order to construct a measure of real output. Of course these prices are producer prices and thus do not correspond in any way to the price of the corresponding imported good. For these situations, the *SNA 93* recommends (paragraph 16.53) using average price changes of similar products as a proxy for the change in price of a new good between the base period and the current period. We interpret this recommendation by SNA in the context of our model by using what *would have been* the base period price for a good produced in period t by a country *if the country had produced* that good in the base period. So, for example, we value a good z produced by country 1 in period t but not in period 0 by the price $q_0^1(z)$, the price at which good z would have sold, had it been produced by country 1 in period 0.

In addition to GDP, we are interested in looking at the model's predictions for aggregate TFP comovements. We define aggregate TFP by the formula

$$TFP_t^i = \frac{\hat{Y}_t^i}{\left(\hat{K}_t^i\right)^{\alpha}\left(L_t^i\right)^{1-\alpha}} \,. \tag{1.18}$$

We compute TFP as the Solow residual, so that it is comparable with estimates of TFP processes used in the international business cycles literature. In light of the data presented earlier, accounting for TFP correlations increasing with trade intensity is an important factor in resolving the trade-comovement puzzle.

Labor is in the same units for all periods, but capital, since it is cumulated from investment expenditures on different goods, raises the same measurement issues as GDP. For comparability with standard growth accounting practices, we construct real investment as the model's aggregate investment expenditures divided by the GDP deflator implied by the definitions in (1.16) and (1.17),

$$\hat{X}_t^i = \int_0^1 p_t^i(z) x_t^i(z) \times \frac{\hat{Y}_t^i}{Y_t^i} \,,$$

and construct the capital stock using an analogue of the model's capital law of motion (1.4):

$$\hat{K}_{t+1}^i = (1-\delta)\hat{K}_t^i + \hat{X}_t^i \,.$$

The initial capital stocks, $\hat{K}_0^1$ and $\hat{K}_0^2$, are chosen so that the growth rate of the capital stock in country 1 from period 0 to period 1 equals the average growth rate of the capital stock for the rest of the time horizon, and the two countries are symmetric in the initial period.

1.3.6 International Trade and Cross-Country comovement

In this section we briefly explain the two mechanisms through which the model has the potential to generate positive cross-country correlations between countries' GDP, as well as higher correlations for more significant trading relationships.

First, the presence of endogenous trade patterns allows for a larger set of goods being traded in equilibrium among closer trading partners. An aggregate shock to one country will increase the demand coming from that country. Lower tariff barriers will imply that a larger range of goods is traded. Therefore the higher demand in one country can be transmitted through a larger set of foreign goods. The adjustment of trade through the extensive margin of goods traded is absent in models that feature the Armington aggregator, where adjustment comes only through a higher demand over a fixed range of goods.

Second, the presence of tradeable intermediate goods, in conjunction with the endogenously determined patterns of trade in equilibrium, has the potential to transmit aggregate productivity shocks between countries. For example, if one country receives a favorable productivity shock, it's industries' demand for intermediate goods increases. Lower tariff barriers imply a larger share of intermediate goods traded in equilibrium. This will tend to raise GDP in the second country. In addition, since some of the first country's output of goods is also used as intermediates, the increase in productivity makes these goods cheaper to import for the second country, so that the second country becomes more productive at producing output.

1.3.7 Correlation of TFP

Having explained how we compute the model statistics and the intuition of the channels of comovement of output present in our model, we will analyze our main quantitative finding. In particular even though we find that versions of our model can qualitatively account for the trade-comovement puzzle, no version can quantitatively solve it. The reason is that there is no intrinsic relationship in the model linking levels of trade with TFP comovement except for the relationship of the comovement linked with the tariff revenue part of measured TFP (see expression (1.17) and (1.18)). In fact, in the appendix of this paper we look at the log-linearized expression for gross value added (GDP excluding tariffs) and TFP. We find that for the Cobb-Douglas case any change in the measured TFP in the model is only (at least to a first-order approximation) due to the assumed correlation in the variation of exogenous technology shocks plus any resulting correlation related to the the tariff revenue of the two model economies. The intuition is similar to the one stated by Bernard et al. (2003) for the link between efficiency and measured productivity: under perfect competition and constant returns to scale technology, each sector employs inputs in the same proportion. Thus reallocation of production towards more productive sectors in response to technology shocks would not appear as increases in measured productivity.[8]

We have set up the model in a way that it could potentially generate measured TFP dependence on trade. However, when accounting for measured TFP as is done in the data, measured TFP is equal to the aggregate productivity shock plus the part related to the tariff revenue. Thus, the patterns of trade specialization do not influence in any quantitatively relevant way the measured TFP correlations.

The tariff revenue for each country as a function of the unilateral tariff rate resembles a Laffer curve, namely it is initially an increasing and eventually a decreasing function. Given that we assume balanced trade the tariff revenue is perfectly correlated among the

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[8] Even for production specifications different than the Cobb-Douglas one, our performed simulations do not deliver a positive relationship between trade and correlations of measured TFP's.

two countries. As a consequence if the fraction of total tariff revenue as a fraction of GDP is high, then the correlation of tariff revenue can substantially influence the comovement of measured GDP and appear in the measured TFP as well. This will show up in some of our experiments. Of course, the GDP correlation will also be affected by the correlation of the labor and capital of the two countries. In the next section we quantitatively assess all of the channels of transmission of aggregate GDP.

1.4 Results

1.4.1 Parameter Values

A restriction in our analysis is the two country context we assume. To perform a complete quantitative analysis of the trade-comovement puzzle a three country context has to be employed as in Kose and Yi (2006). However, our objective is to identify whether two prominent mechanisms of international trade theory, namely endogenous specialization and intermediate goods, can lead to a resolution of the puzzle. These mechanisms have been suggested that have been suggested as possible solutions of the trade-comovement puzzle (see for example Kose and Yi (2003) and Burstein et al (2004)).[9] Therefore, we aim to present model results in a two country context with a plausible set of parameters rather than a full calibration. For the productivity schedules $A_t^i(z)$, recall that we assume that

$$A_t^i(z) = A_t^i \varphi^i(z)$$

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[9] However, Burstein et al (2004) suggest the possibility that domestic and foreign intermediate inputs are very complementary to production. In the future, we plan to consider the case of foreign and domestic inputs being more complementary in production than foreign and domestic final good.

with the functions φ^i specified as:

$$\begin{aligned}\varphi^1(z) &= 1+\lambda(1-z)\ ,\\ \varphi^2(z) &= 1+\lambda z\ .\end{aligned}$$

This formulation contains the simplification that aggregate shocks to technology affect all industries in the same way. However, it simplifies parameter selection, as we only need a time-varying process for the aggregate shock and not for the entire schedule of productivities.

We assume that the aggregate technology shocks follow an AR (1) process in logarithms:

$$\log\begin{bmatrix} A^1_{t+1} \\ A^2_{t+1}\end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22}\end{bmatrix}\log\begin{bmatrix} A^1_t \\ A^2_t\end{bmatrix} + \begin{bmatrix} \varepsilon^1_{t+1} \\ \varepsilon^2_{t+1}\end{bmatrix}\ .$$

The numbers a_{ij} are parameters, and $[\varepsilon^1_t, \varepsilon^2_t]$ is an i.i.d. mean-zero normal random variable. We use the following parameters: $a_{11} = a_{22} = 0.9$, $a_{12} = a_{21} = 0$, $stdev(\varepsilon^i_t) = 0.008$, $cov(\varepsilon^1_t, \varepsilon^2_t) = 0.000016$. These parameters imply that $corr(\varepsilon^1_t, \varepsilon^2_t) = 0.25$, a standard value in the international business cycles literature. We set $a_{12} = a_{21} = 0$, not allowing for spillover of the TFP shocks, in order to isolate the effect that our mechanism has in the comovement of aggregate variables. However, allowing for positive correlation of the TFP shocks does not distort our results. It only changes in the same magnitude the levels of the correlations of aggregate variables across all simulations.

The rest of the parameters of the model are standard in the international real business cycles literature. We set these parameters given that we interpret one model period as one year. We assign α a value of 0.3, implying that 30% of income from value added is paid to capital services. The share of intermediate goods in gross output, θ, is set to 0.5. The depreciation rate δ is set to 5% per period. The discount factor β is set to 0.96, so that the steady state rate of return on capital is 4% per period. The utility parameter μ

is set to 0.34, implying that approximately 30% of the total time endowment is spent on labor in the steady state. The intertemporal substitution parameter σ is set to 2, and the parameter governing the elasticity of substitution between goods from different industries, ρ, is set to 0.33 implying an elasticity of substitution for different industries' goods of 1.5. The parameter λ, which determines the slopes of each country's productivity curves $\varphi^i(z)$, is set to 2. The tariff rate τ is also varied within a range, as described later below.

The parameter γ, that determines the share of tradeable goods in consumption and investment is calibrated differently for the experiments we perform. In particular, we consider a case of a large import to GDP ratio trade relationship. In this case the benchmark model is calibrated to generate a 23% imports/GDP ratio with a 15% tariff. We interpret the 15% tariff rate as the average tariff rate in the developed countries in the 1980s and 1990s, motivated by empirical evidence on tariff rates summarized by Kose and Yi (2006). The 23% imports/GDP ratio corresponds to average imports to GDP ratio for the countries in our sample (excluding Belgium-Luxembourg) for the years that our data sample runs (1971-2002). We also run an experiment where the imports/GDP ratio corresponds to the average imports to GDP ratio in the sample of the 21 countries we use for a tariff level of 15%. In this case we calibrate all the model to a .5% imports/GDP ratio.

1.4.2 Specifying a Range of Tariffs

We perform two different experiments. The first, indicated in Tables 1.2 and 1.3 refers to a bilateral trade relationship where the imports/GDP ratio is large and the second, in Tables ?? and ?? refers to a bilateral trade where the imports/GDP ratio is small. To look at the dependence of the correlations on the intensity of the trade relationship, we generate a cross-section of observations by varying the uniform tariff rate between 0 and 0.38. We choose this range of tariffs which is similar to Kose and Yi (2006) simply to create 20 equally spaced tariff intervals. The model economies with different tariff rates are considered as representing distinct pairs of countries. We look at the dependence of GDP and TFP correlations for each set of trading partners on the intensity of the trade relationship. As we did with the actual cross-section data, we run simple regressions of each correlation on the degree of trade intensity. The latter is measured as the log of the average over time periods of the following model statistic:

$$\frac{\int_0^{z_t^\ell} p_t^2(z)\left(c_t^2(z)+x_t^2(z)+\int_0^1 m_t^2(\zeta,z)d\zeta\right)dz+\int_{z_t^h}^1 p_t^1(z)\left(c_t^1(z)+x_t^1(z)+\int_0^1 m_t^1(\zeta,z)d\zeta\right)dz}{Y_t^1+Y_t^2}$$

The numerator in the above expression is simply the sum of the imports, valued at purchaser's prices, into country 2 from country 1, and into country 1 from country 2. The denominator is the sum of the two countries' GDPs (Y_t^i was defined in (1.16)).

We are primarily interested in the coefficient determining the dependence of either of the correlations on trade intensity. For comparison with the data, recall that the regression coefficient for GDP correlation on trade intensity was 0.094 and the coefficient for TFP correlation was 0.055. We test four versions of our model, the benchmark model described above, the model with no intermediate inputs, and our version of the Armington aggregator with and without intermediate inputs. For the first quantitative experiment with high import/GDP ratios we report the results in Tables 1.2 and 1.3. For this expreriment, in the first two models we pick the nontradable share to generate a 23% imports/GDP ratio with a 15% tariff. The models with Armington aggregator have fixed specialization cutoffs throughout all experiments and thus $z_t^l = z_{ss}^l$, $z_t^h = z_{ss}^h$. We pick z_{ss}^l and z_{ss}^l for the Armington aggregator model with and without intermediate goods to be equal to the ones of the endogenous specialization model with and without intermediates respectively when they are calibrated to match the a 23% imports/GDP ratio with a 15% tariff.

The results for the "benchmark" model are reported in the first column of Tables 1.2 and 1.3. The slope is positive but quantitatively very small. Considering the coefficient of the regression for higher level of tariffs reveals a larger coefficient, though still not close to the one observed in the data. The low coefficients are partially due to the "Laffer curve" effect which strong and in the wrong direction especially for low tariff levels. This can be seen from the slope of the measured TFP regression that should represent only higher correlation due to higher correlation of the tariff revenue according to what was argued in the previous section. A noticeable fact is that the correlation of labor and investment goes in the right direction. In particular, the coefficient of the regression of correlation of labor to the logarithm of trade intensity is .076 for investment and .119 for labor. This implies that the correlation of labor and investment increases with the right magnitude with trade, but it is not translated to transmission of measured GDP correlation. The "Laffer-curve" effect and the fact that increases in trade are not related with aggregate productivity cancels out the effects of the increase in correlation of labor and investment. For the version of the model with no intermediate goods ($\theta = 1$), the implied slopes are slightly negative. To this point we find that for similar trade (rather than tariff) levels the models with and without intermediate goods look very similar.[10] However, for the Armington aggregator version of our models with or without intermediate goods the slope of the regression is significantly more negative. In this version of our model the tariff at any time is applied over a given set of goods and thus the tariff revenue is always of a substantial magnitude. Given the "Laffer-curve" effect this makes the overall correlation of GDP to have the wrong sign. Below, we will also look at the performance of the endogenous specialization and Armington aggregator models by adjusting for the "Laffer-curve" effect.

In the second experiment reported in Tables 1.2 and 1.3 we calibrate the models in a similar manner as in experiment 1 except that we pick the parameter γ so that the models with endogenous specialization generate .5% imports/GDP ratio with a 15% tariff. We see that the patterns are similar to the previous experiment, but the quantitative magnitudes

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[10] In the future, we plan to quantitatively evaluate the mechanism suggested by Burstein et al (2004) namely that intermediate inputs have a lower elasticity of substitution than final goods.

are very small. The reason of course is that due to the small trade between countries, any propagation of the shocks from the one country to other is small comparing to the GDP of the country. Thus, the propagation of the shocks through trade is relatively weak.

Finally, we would like to see whether endogenous specialization can potentially be a quantitatively important channel for the transmission of output shocks to different countries. We perform a decomposition of the change in correlation of GDP and and Gross Value Added (GDP excluding tariff revenue) due to different trade intensity levels to see if adjustments in the extensive margin of trade (present only in the endogenous specialization model) or adjustments in the intensive margin of trade are more important for the transmission of output shocks. In Table 1.5 we present the results for the high imports/GDP experiment (23% under **free trade**) and Table 1.6 for low imports/GDP experiment (.5% under **free trade**).[11] For either case, we vary trade intensity by changing the tariff and allowing the set of traded goods to change. We find that in the endogenous specialization model the correlation of the Gross Value Added (GVA) is lower with smaller trade (the GDP correlation is higher due to the tariff effect). To decompose the effects of the change in trade intensity on the correlations, we then look at the correlation of GDP and GVA when trade intensity changes in two cases using the Armington aggregator version of our model. In the first case we change only the cutoffs of trade (extensive margin) but do not change the tariffs, and in the second we change the tariffs (thus, there is adjustment in the intensive margin) but not the cuttoffs. We see that in the case with changing cutoffs the implied slope for the correlation of GVA is bigger. We interpret these results as evidence that adjustments on the extensive margin of goods traded–present only in the endogenous specialization model–can partially explain the inability of Armington aggregator models to account for the trade co-movement puzzle. However, given the overall inability of the model to achieve the quantitative size of the correlations, the endogenous specializa-

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[11] In this experiment, and in order to facilitate exposition, the models have been calibrated to generate a 23% and .5% imports/GDP ratio under free trade rather under a 15% tariff rate.

tion mechanism does not contribute substantially towards the quantitative resolution of the trade-comovement puzzle.

1.5 Conclusion

In this paper we investigate the potential ability of two prominent mechanisms of international trade theory to explain the trade-comovement puzzle reported by Kose and Yi (2001, 2006). We parsimoniously model the mechanisms of endogenous specialization and intermediate goods in a framework which encapsulates the standard international business cycles model. Our main finding is that, when accounting for GDP as measured in the data, neither the previous models using the Armington aggregator as in Backus et al (1994), nor our additions to the standard model can quantitatively generate the observed dependence of correlations of GDP and trade. We present data that indicate the dependence of correlations of measured TFP and trade. We show that none of the abovementioned versions of our model can account for the dependence of measured TFP and trade. We conclude that future research has to address this particular observation in order to resolve the trade-comovement puzzle.

A channel that remains unexplored in our analysis is the one suggested by Burstein et al (2004), namely that intermediate inputs have a lower elasticity of substitution than final goods. This extension of our model is left for future investigation.

1.6 Appendix

1.6.1 Measured TFP in the Model

We derive the equality between measured TFP and the exogenous technology shock, when tariff revenue is not included in GDP, for simplicity of exposition.

The gross output in country 1 is

$$\widehat{GO_t^1} = \int_{\Omega_t^1} q_0^1(z)\left(\frac{w_t^1\ell_t^1(z) + r_t^1 k_t^1(z) + P_t^1 M_t^1(z)}{q_t^1(z)}\right) dz\ .$$

We consider the case where the production function is Cobb-Douglas, but the numerical simulations reveal very similar patterns for all the other cases as well. The producer price $q_t^i(z)$ is the product of a time-varying aggregate part and a constant individual part that is related to the relative productivity of each sector,

$$q_t^i(z) = \left(\frac{w_t^1}{\theta(1-\alpha)}\right)^{\theta(1-\alpha)}\left(\frac{r_t^1}{\theta\alpha}\right)^{\theta\alpha}\left(\frac{P_t^1}{1-\theta}\right)^{1-\theta}\left(A_t^1\right)^{-\theta}\varphi^1(z)^{-\theta}$$

Therefore, the constant individual part cancels out and the above expression for gross output is:

$$\begin{aligned}\widehat{GO_t^1} &= \int_{\Omega_t^1}\frac{q_0^1}{q_t^1}\left(w_t^1\ell_t^1(z) + r_t^1 k_t^1(z) + P_t^1 M_t^1(z)\right) dz\\ &= \frac{q_0^1}{q_t^1}\left(w_t^1 L_t^1 + r_t^1 K_t^1 + P_t^1 M_t^1\right)\end{aligned}$$

Denoting by ˜ the log-linearized variables we have that using the above two relationships

$$\widetilde{\widehat{GO_t^1}} = \theta\widetilde{A_t^1} + \theta(1-\alpha)\widetilde{L_t^1} + \theta\alpha\widetilde{K_t^1} + (1-\theta)\widetilde{M_t^1} \tag{1.19}$$

The intermediate consumption is given by the following expression

$$\widehat{IC_t^1} = M_t^1 \int_0^1 p_0^1(\zeta) \left(\frac{p_t^1(\zeta)}{P_t^1} \right)^{1/(\rho-1)} d\zeta$$

more analytical and given the accounting of the model explained in the main text

$$\widehat{IC_t^1} = M_t^1 \left(\frac{1}{P_t^1} \right)^{1/(\rho-1)} \left[\begin{array}{c} \int_{[0,z_0^h]\cap[0,z_t^h]} q_0^1(\zeta)\, q_t^1(\zeta)^{1/(\rho-1)} d\zeta \\ +(1+\tau)^{1/(\rho-1)} \int_{[0,z_0^h]\cap[z_t^h,1]} q_0^1(\zeta)\, q_t^2(\zeta)^{1/(\rho-1)} d\zeta \\ +(1+\tau) \int_{[z_0^h,1]\cap[0,z_t^h]} q_0^2(\zeta)\, q_t^1(\zeta)^{1/(\rho-1)} d\zeta \\ +(1+\tau)^{\rho/(\rho-1)} \int_{[z_0^h,1]\cap[z_t^h,1]} q_0^2(\zeta)\, q_t^2(\zeta)^{1/(\rho-1)} d\zeta \end{array} \right]$$

We suppose that country 2 receives a favorable technology shock. Other cases can be handled similarly. A favorable shock to country 2 increases A_t^2, and z^ℓ, z^h both decrease from period-0 values. Therefore $[0,z_0^h]\cap[0,z_t^h] = [0,z_t^h]$, $[0,z_0^h]\cap[z_t^h,1] = [z_t^h, z_0^h]$, $[z_0^h,1]\cap[0,z_t^h] = \emptyset$ and $[z_0^h,1]\cap[z_t^h,1] = [z_0^h,1]$. The above expression can be re-written

$$\widehat{IC_t^1} = M_t^1 \left(\frac{1}{P_t^1} \right)^{1/(\rho-1)} \left[\begin{array}{c} q_0^1 \left(q_t^1\right)^{1/(\rho-1)} \int_0^{z_t^h} \varphi^1(\zeta)^{\rho/(\rho-1)} d\zeta \\ +(1+\tau)^{1/(\rho-1)} q_0^1 \left(q_t^2\right)^{1/(\rho-1)} \int_{z_t^h}^{z_0^h} \varphi^1(\zeta)\, \varphi^2(\zeta)^{1/(\rho-1)} d\zeta \\ +(1+\tau)^{\rho/(\rho-1)} q_0^2 \left(q_t^2\right)^{1/(\rho-1)} \int_{z_0^h}^1 \varphi^2(\zeta)^{\rho/(\rho-1)} d\zeta \end{array} \right]$$

After log-linearizing, using the symmetry in the steady state which implies $q_0^1 = q_0^2$, and

$$\left(P_t^1\right)^{\rho/(\rho-1)} = \left(q_t^1\right)^{\rho/(\rho-1)} \int_0^{z_t^h} \varphi^1(z)^{\rho/(\rho-1)} dz + (1+\tau)^{\rho/(\rho-1)} \left(q_t^2\right)^{\rho/(\rho-1)} \int_{z_t^h}^1 \varphi^2(z)^{\rho/(\rho-1)} dz$$

we get

$$\widetilde{\widehat{IC_t^1}} = \tilde{M}_t^1 \tag{1.20}$$

Therefore, by expressions (1.19), (1.20) and the Cobb-Douglas assumption for the production function we derive that

$$\widetilde{GDP}_t^1 = \frac{1}{\theta}\widetilde{GO}_t^1 - \frac{1-\theta}{\theta}\widetilde{IC}_t^1 = \widetilde{A}_t^1 + (1-\alpha)\,\widetilde{L}_t^1 + \alpha\widetilde{K}_t^1$$

$$\widetilde{TFP}_t^1 = \widetilde{GDP}_t^1 - (1-\alpha)\,\widetilde{L}_t^1 - \alpha\widetilde{K}_t^1 = \widetilde{A}_t^1$$

1.6.2 Description of Cross-Section Data

Year covered: 1971-2002

Countries Covered:Australia, Austria, Belgium (with Luxembourg), Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Norway, Netherlands, New Zealand, Portugal, Spain, Sweden, Switzerland, United Kingdom, United States.

Data Sources

Trade data are extracted from the International Monetary Fund's Direction of Trade Statistics . We have constructed the exports using imports so that $X_{ijt} = M_{ijt}$. In this way we avoid problems of mis-report of export data and we use the most reliable reported data on imports. Real GDP and Real gross capital formation data are taken from the World Development Indicators data of the World Bank. We construct capital using the perpetual inventory method. The initial capital stock is chosen so that its growth rate from 1971 to 1972 can match the growth rate of the capital for the rest of the time horizon. Employment data are from OECD.

Solow residuals are constructed for each country using the formula $A_t = \frac{Y_t}{(K_t)^{\alpha}(L_t)^{1-\alpha}}$ where Y_t, L_t are directly from the data and K_t is constructed as mentioned above. We set $\alpha = 0.3$ which as Gollin (2002) suggests is a reasonable estimate for the capital share in production for many countries. We detrend the series of A_t, Y_t after taking logarithms using the HP-filter.

Measure of Bilateral trade intensity: $\log \frac{\text{imports+partner's imports}}{\text{GDP+partner's GDP}}$.

Descriptive statistics

Statistics of the sample (210 observations):

Variable	Average	Median	Stand. dev.
trade intensity	0.005	.002	0.008
GDP correlation	0.352	0.389	0.307
TFP correlation	0.277	0.304	0.255

Note: We also run the regression for the years 83-02 and 93-02, with results very similar to the ones mentioned in the text.

Table 1.1: Data cross-section regression

1970-2002	GDP	TFP
trade intensity	0.094 (0.016)	0.055 (0.014)
constant	0.915 (0.099)	0.605 (0.09)
R^2	0.137	0.067

Table 1.2: Model cross-section GDP regression, varying tariff (high imports/GDP)

regression coefficient and R^2	Bench.	Bench., No Int	Arm.	Arm., No Int
trade intensity	.004	-.022	-.138	-.185
R^2	.17	.77	.99	.99
trade intensity (τ =16%-38%)	.016	-.004	-.119	-.160
R^2	.99	.37	1.00	1.00

Table 1.3: Model cross-section TFP regression, varying tariff (high imports/GDP)

regression coefficient and R^2	Bench.	Bench., No Int	Arm.	Arm., No Int
trade intensity	-.028	-.102	-.207	-.302
R^2	.64	.94	.99	1.00
trade intensity (τ =16%-38%)	.002	-.063	-.181	-.270
R^2	.04	.97	1.00	1.00

Table 1.4: Model cross-section regression, varying tariff (low imports/GDP)

regression coefficient and R^2	Bench.	Bench., No Int	Arm.	Arm., No Int
trade intensity	.0001	-.0005	-.0031	-.0043
R^2	.15	.77	.99	.99
trade intensity (τ =16%-38%)	.0004	.0000	-.0027	-.0037
R^2	.99	.39	1.00	1.00

Table 1.5: Model implied slopes, varying tariffs or cuttoffs (high imports/GDP)

	trade int.	corr. GDP	corr. GVA	slope GVA
Free trade	.230	.326	.326	-
End. Spec.: $\tau = .15$	.168	.344	.308	.059
$\tau = 0, z^\ell = .43, z^h = .57$	.201	.315	.315	.080
$\tau = 0.15, z^\ell = .5, z^h = .5$	.193	.360	.317	.048

Table 1.6: Model implied slopes, varying tariffs or cuttoffs (low imports/GDP)

	trade int.	corr. GDP	corr. GVA	slope GVA
Free trade	0.00500	0.26134	0.26133	-
End. Spec.: $\tau = .15$	0.00366	0.26177	0.26096	0.00122
$\tau = 0, z^\ell = .43, z^h = .57$	0.00438	0.26112	0.26112	0.00168
$\tau = 0.15, z^\ell = .5, z^h = .5$	0.00420	0.26213	0.26117	0.00098

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Chapter 2

International Trade Dynamics with Intermediate Inputs

2.1 Introduction

This paper builds a model of international trade in intermediate inputs with heterogeneous producers, in which the producer-level decision to use imported inputs is irreversible. The model is used to analyze the dynamic behavior of aggregate and producer-level trade flows in response to movements in the relative price of imported to domestically produced goods. Aggregate trade data show that imports relative to domestic purchases move slowly in response to changes in the relative price of imports. Long-term growth in trade is much larger than the immediate response to trade reform. The model presented here accounts for the slow-moving dynamic behavior of aggregate trade flows, as a result of the irreversiblity in the decision to import intermediate inputs at the micro-level.

Intermediate goods comprise about forty to sixty percent of total international merchandise trade for many of the world's industrial economies.[1] At the micro level, producers

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[1] See Table 2.1 for details.

are heterogeneous in their use of imported relative to domestically produced intermediate inputs. Namely, relatively few producers use imports, and importers are larger in size than non-importers. For example, in both the US and Chile, only about one quarter of manufacturing plants use imported intermediate inputs. In addition, these importing plants employ two to three times as many workers, on average, as their non-importing counterparts.[2] Many empirical studies have documented analogous facts for exporting producers, and most of the theory developed so far incorporating heterogeneity in producer-level participation in international trade has focused on exporting behavior.[3]

This paper instead focuses on the producer-level importing decision to study trade in intermediate inputs, in light of the evidence of the importance of heterogeneity in importing behavior. The importing decision is modeled at the plant level as an irreversible technology choice: a plant can choose a production technology that uses intermediate inputs of only domestically produced goods, or a technology that combines imported and domestic intermediates. The technology that a plant chooses when it is built is fixed for the life of the plant, so the decision to import or not is permanent. This feature of the model is motivated by plant-level evidence. In the data, the plant-level responses to changes in the relative price of imports over time indicate that there is substantial irreversibility in the composition of intermediate inputs that plants use; importing is a relatively irreversible choice.[4]

With plants divided into importers and non-importers based on their initial investment decisions, movements in the relative price of imported to domestic goods affect the volume of aggregate trade through three mechanisms. The first is the within-plant ratio of imports

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[2] See Kurz (2006) for the US, and Section 2 below for Chile. Similar findings are reported in Amiti and Konings (2005) for Indonesia; Biscourp and Kramarz (2006) for France; and Halpern, Koren, and Szeidl (2005) for Hungary.

[3] Empirical studies of exporting behavior include Bernard and Jensen (1995) and Clerides, Lach and Tybout (1998). Theoretical models of exporting behavior include Melitz (2003) and Bernard, Eaton, Jensen and Kortum (2003).

[4] Kasahara (2004), using the same plant data, also finds that a large change in the ratio of imports relative to domestic inputs within a plant is associated with a large concurrent investment in physical capital, interpreted as the adoption of a new technology.

relative to domestic inputs. The second mechanism is the equilibrium allocation of factors of production across existing importing and non-importing plants at any point in time. The third is the dynamic allocation of investment in importing across newly established plants. A decrease in the price of imports relative to domestic goods makes importers relatively more profitable than non-importers. The static effects associated with this change are that importing plants use imports more intensively, and importing plants expand relative to non-importing plants. In addition, if it is expected to persist, the dynamic effect of a price decrease is that newly established plants expect a higher gain in profit from using imports; thus more plants undertake the investment required to import. These two effects determine the response over time of aggregate trade flows to the change in the relative price of imported to domestic goods. Because the dynamic behavior of aggregate imports relative to domestic goods are linked to the rate at which new plants are created, aggregate trade flows respond slowly to changes in the relative price of imports.

The model is calibrated so that both the fraction of plants importing and their size relative to non-importers match the plant-level statistics previously mentioned. The calibrated model is used to measure the contributions of the static and dynamic reallocation effects to the short-run and long-run dynamics of aggregate trade flows. When the model is subjected to aggregate technology shocks of standard business cycle magnitudes, the static effect is predominant. This is because new plants are a small fraction of the total. The model predicts fluctuations in aggregate trade flows that are characterized by a low elasticity of substitution between imported and domestic intermediate goods. A permanent trade liberalization, however, is followed by a large, gradual increase in the volume of trade over several years following the policy change. The number of importing plants relative to non-importing plants increases over time. In response to a trade reform of reasonable magnitude, the model predicts a long-run doubling in the volume of trade relative to GDP, with about half the growth in trade occurring within ten years.

This paper is related to recent work on dynamic models of producer-level exporting decisions. These include Ruhl (2005), Ghironi and Melitz (2005), Alessandria and Choi (2005 and 2006), and Atkeson and Burstein (2006). As in Ruhl (2005), this paper isolates different effects that influence the short-run and long-run response of trade flows to relative price changes. Ghironi and Melitz (2005) and Alessandria and Choi (2005) examine the business cycle properties of models with fixed costs of exporting. Alessandria and Choi (2006) and Atkeson and Burstein (2006) study the transition path following trade liberalization in models in which producer-level efficiency evolves over time.[5] In contrast, in the model of importing behavior presented here, cyclical fluctuations in trade flows and gradual growth in trade depend on the irreversibility of the choice between importing and non-importing technologies. The models of exporting in previous studies differ in the extent to which the decision to export is irreversible.[6] However, they all share the feature that the decision made at any time to *not* export can be undone. The essential difference between the model in this paper and previous models of dynamic exporting decisions is that, in this paper, *either* of the choices available to producers - to not import or to import - is a permanent decision.

The assumption of irreversibility in technology choice is similar to that in models of "putty-clay" capital, recent examples of which include Atkeson and Kehoe (1999) and Gilchrist and Williams (2000). In these models, investing in capital requires an irreversible choice of the amount of another variable input that will be combined with the capital in the future. (The variable input is energy in Atkeson and Kehoe (1999) and labor in Gilchrist and Williams (2000)). The application of this type of irreversibility to production with imported and domestic intermediate inputs in this paper is motivated by Kasahara (2004),

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[5] Chaney (2005) also considers the transition path following trade reform in a model with producer-level exporting decisions, but focuses on the average productivity of operating plants rather than the behavior of trade flows.

[6] In Ruhl (2005), the decision to export is completely irreversible. In Ghironi and Melitz (2005) the decision is made independently each period. Alessandria and Choi (2005) incorporate both irreversible and independent per-period dimensions in the decision to export.

who finds evidence of the putty-clay nature of a producer's choice between imported and domestic intermediate goods.

A recent paper on producer-level importing decisions is Kasahara and Lapham (2006), who consider a producer's joint import and export decisions in a stationary model derived from that of Melitz (2003). Their model incorporates fixed costs of importing to generate cross-sectional differences in the use of imports by plants. This paper analyzes an environment with aggregate dynamics, and finds that the irreversibility in individual plant technology and the cross-section heterogeneity associated with fixed costs of importing can account well for the dynamic behavior of trade flows observed in the data.

The rest of the paper is organized as follows. Section 2 presents data for the aggregate and plant-level facts mentioned in this introduction. Section 3 presents the model and characerizes the plant-level and aggregate implications of relative price movements. Section 4 provides a calibration and quantitative analysis of the model, and Section 5 concludes.

2.2 Data

This section presents two sets of facts from the data that motivate the paper. The first set of facts, from aggregate trade data, establishes that the response of trade flows at the aggregate level responds slowly to changes in relative prices across countries. The second set of facts provides plant-level evidence that on the costly and irreversible aspects of the decision to use imported intermediate inputs, and therefore motivates the approach taken in this paper in accounting for the observations in the aggregate data.

2.2.1 Aggregate Facts

Sudden changes in the price of imported goods have gradual effects on a country's imports. Figure 2.1 depicts the total imports by Mexico from the United States, relative to US GDP, over the period 1982-2000, along with average Mexican tariffs on US goods.[7] During this period, there were two episodes in which tariffs were reduced by a large amount within a single year: Mexico's unilateral trade liberalization in 1988, and the regional North American Free Trade Agreement with the US and Canada in 1994. There was substantial growth in trade over this period, with imports from the US relative to US GDP growing four-fold from 1987-1993 and nearly doubling again from 1993-2000.

Attributing the growth in Mexico's trade with the US to the large tariff cuts in 1987 and 1993 implies that changes in the price of imported relative to domestic goods generate large changes in trade flows. However, the growth in trade from a one-time tariff reduction is gradual, slowly accumulating over several years.

Another way to depict the gradual response of trade flows to price changes is the "elasticity puzzle" described in Ruhl (2005). Researchers estimating the elasticity of substitution between imported and domestic goods rely on either business cycle fluctuations, or on single trade liberalization events, to generate variation in the price of imports realtive to domestic goods. The estimates from cyclical fluctuations in prices imply small elasticities, mostly in the range of 1-2, while estimates from the growth in trade several years following trade liberalizations imply large elasticities, generally above 6. Therefore, the response in trade growth to a price change takes time to develop.

2.2.2 Plant-level Facts

This section describes data from a panel survey of Chilean manufacturing plants, from Chile's *Instituto Nacional de Estadistica (INE)*. The period covered is 1979-1986. Each plant reports its imported and total intermediate input purchases. If imports are positive, I consider the plant an importer.

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[7] Trade and GDP data are from International Monetary Fund, *International Financial Statistics* CD-ROM. Mexican tariffs are from Hinojosa-Ojeda et al. (2000) for 1982-1994, and from Office of US Trade Representative, *Trade Policy Agenda* and *Report on Foreign Trade Barriers* (various years).

Cross-section

I first describe the cross-section characteristics of plants. Statistics are computed for all plants existing in the sample in each year, then averaged across years.

Few manufacturing plants in Chile use imported intermediate inputs, and they tend to be much larger than the plants that do not use any imported inputs. Table 2.2 shows that only about 24 percent of plants, on average, use a positive amount of imported intermediate inputs. These plants employ about three times as many workers, on average, as the plants that do not use imported inputs.

For comparison, Kurz (2006) reports that in 1992, about the same proportion of US manufacturing plants use imported inputs, and they are on average about twice the size of the plants that do not.

These figures imply that using imported inputs along with domestic inputs is disproportionately more costly than using domestic inputs alone. In addition, Kasahara and Rodrigue (2005), using the same sample of Chilean plants, find that using imported along with domestic inputs brings with it a significant gain in plant productivity, so that plants operating at a larger scale would benefit the most from using imports. Therefore, only large plants find it worthwhile to pay the additional costs of using imported inputs.

Panel

The allocation of resources across plants over time provides evidence that the decision to use imported inputs or domestic inputs alone is not easily reversed. Over the period 1979-1986, the aggregate quantity of imported relative to total intermediate inputs purchased by Chilean manufacturing plants declined by 18 percent per year, on average.

In light of the cross-section heterogeneity among plants' use of imports highlighted in the previous subsection, this aggregate decline can be attributed at the plant-level to several different channels. If some plants import and some do not, and plants can enter and exit the economy, aggregate imports relative to total intermediate inputs can fall because: (i) importing plants import relatively less of their inputs; (ii) importing plants shrink relative to non-importing plants; (iii) importing plants stop importing and become non-importing plants; or (iv) importing plants that exit the economy are replaced by entering plants that do not import.

Magnitudes can be assigned to these channels through decomposing the aggregate ratio of imported to total intermediate inputs as follows. Let $M_t = \sum_{i \in I_{mt}} m_t^i$ be the aggregate quantity, in year t, of imported inputs used at importing plants, where i denotes a plant, m_t^i denotes imported inputs used by plant i in year t, and I_{mt} is the set of plants that uses imports in year t. Similarly, let $X_t = \sum_{i \in I_t} x_t^i$ be the aggregate quantity of total intermediate inputs (imported plus domestic) used by all plants, with x_t^i denoting all the intermediate inputs purchased by plant i in year t, and I_t denoting the entire set of plants operating in period t.[8] Then, the change at the aggregate level in imports relative to total intermediate goods can be decomposed as follows:[9]

$$\begin{aligned}
\frac{M_{t+1}}{X_{t+1}} - \frac{M_t}{X_t} = & \sum_{i \in I_{mt+1} \cap I_{mt}} \frac{x_t^i}{X_t} \left(\frac{m_{t+1}^i}{x_{t+1}^i} - \frac{m_t^i}{x_t^i} \right) \\
& + \sum_{i \in I_{mt+1} \cap I_{mt}} \left(\frac{x_{t+1}^i}{X_{t+1}} - \frac{x_t^i}{X_t} \right) \frac{m_t^i}{x_t^i} \\
& + \sum_{i \in I_{mt+1} \cap I_{mt}} \left(\frac{x_{t+1}^i}{X_{t+1}} - \frac{x_t^i}{X_t} \right) \left(\frac{m_{t+1}^i}{x_{t+1}^i} - \frac{m_t^i}{x_t^i} \right) \\
& + \sum_{i \in (I_{mt+1} \setminus I_{mt}) \cap (I_t \cap I_{t+1})} \frac{x_{t+1}^i}{X_{t+1}} \frac{m_{t+1}^i}{x_{t+1}^i} - \sum_{i \in (I_{mt} \setminus I_{mt+1}) \cap (I_t \cap I_{t+1})} \frac{x_t^i}{X_t} \frac{m_t^i}{x_t^i} \\
& + \sum_{i \in (I_{mt+1} \setminus I_{mt}) \cap (I_{t+1} \setminus I_t)} \frac{x_{t+1}^i}{X_{t+1}} \frac{m_{t+1}^i}{x_{t+1}^i} - \sum_{i \in I_{mt} \setminus I_{mt+1} \cap (I_t \setminus I_{t+1})} \frac{x_t^i}{X_t} \frac{m_t^i}{x_t^i}
\end{aligned} \tag{2.1}$$

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[8] Total intermediate inputs are deflated with industry-specific input price indices, and imported intermediate inputs are deflated with an economy-wide import price index.

[9] This is similar to the methodologies used by many authors to decompose aggregate productivity growth into its plant-level components. See, for example, Baily, Hulten and Campbell (1992).

The first line in the sum above gives the total effect of each plant that imports in both years t and $t+1$ adjusting its ratio of imported to domestic inputs (m/x), weighted by its initial share in the aggregate economy (x/X). This is adjustment within the plant. The second line is the sum of changes in these continuously importing plants' share of the economy, holding fixed the intensity with which each plant uses imports. This is adjustment by reallocating between plants. The third line gives the effect of the plants' ratios m/x and their shares of the economy x/X changing together. The fourth line is the contribution of continuing plants that start to import in year $t+1$, net of the loss due to continuing plants that no longer import in year $t+1$. Finally, the fifth line is the contribution of new entrants that import less the loss due to importing plants that exit the economy. Table 2.3 gives the contributions of each of these five components, labeled "within", "between", "cross", "switch" and "entry", respectively, as a percentage of the aggregate change $M_{t+1}/X_{t+1} - M_t/X_t$ (so that the components sum to one hundred). Two sets of figures are reported: the average across one-year changes, and the 7-year change.

The figures in the first row of Table 2.3 show that, on average, each year, 78 percent of the decline in imports at the aggregate level is accounted for by each importing plant adjusting the ratio of imports relative to total intermediate inputs it uses. About 26 percent is accounted for by importing plants shrinking in scale relative to non-importing plants. Two percent of the aggregate change is accounted for by new entrants using less imports than exiting plants, and about three percent is attributed to importing plants switching to becoming non-importers more often than non-importing plants switching to importing. The fact that the "between" component is substantial provides evidence that there is some irreversibility in the nature of the decision to import: not all the adjustment at the aggregate level comes from each plant changing the composition of goods it uses. In addition, the year-to-year net effects of entry and exit and of plants switching importing status are very small. In contrast, over the entire 7-year period, the effects of entry and exit accumulate, and contribute five times more to the aggregate change in imports than they do on average each year.

In the model presented in the next section, plants face a costly, irreversible decision to use imported intermediate inputs. This generates both the cross-sectional properties of plant heterogeneity discussed in the previous subsection, and generates trade growth at the aggregate level through the "within", "between", and "entry" plant-level margins discussed here. When calibrated to match the cross-sectional properties of the plant data, the model generates aggregate implications for the dynamic behavior of trade flows that mimic the aggregate facts discussed earlier in this section.

2.3 Model

2.3.1 Outline

The model economy consists of two countries, referred to as *home* and *foreign*. There are two goods in the economy, and each good is produced in only one country and can be traded internationally. Production in each country is carried out in plants that can operate one of two available technologies to produce their country's good. The first technology combines labor with intermediate inputs of the domestically-produced good. The second technology uses, in addition, intermediate inputs of the imported good. Plants that operate each technology are referred to as non-importing and importing plants, respectively. Plants in the economy are distinguished by the technology they use (denoted d using only domestic goods and m using imports) and the idiosyncratic efficiency, denoted z, with which they operate the technology. All plants are subject to country-wide shocks to aggregate efficiency, denoted A in the home country and A^* in the foreign country. (Throughout, all foreign variables are indexed with an asterisk (*).)

Each period, all plants face a constant probability of death. New plants continually enter the economy and choose the technology, importing or not, with which they will operate. This is an irreversible decision, fixed over the life of each plant. The entry and technology choices of a plant require fixed investment costs that cannot be recovered.

Each country is populated by a continuum of mass one of identical infinitely-lived consumers who are each endowed with 1 unit of time to be allocated between labor and leisure, and an equal share of ownership of the all the plants in the country. The consumers' labor is used for production in all existing domestic plants.

Consumers in each country do not value consumption of the good produced abroad, so there is no trade in goods for final consumption. Output produced in each country is allocated to final domestic consumption, intermediate consumption of domestic and foreign plants, and investment in new plants.

2.3.2 Time and Uncertainty

Time is discrete and indexed $t = 0, 1, \ldots$. At each date t, an event s_t occurs, which is drawn from a Markov process with transition function $\phi\left(s_t|s_{t-1}\right)$. The state of the economy at any date t is the complete history of events up to and including date t, denoted $s^t = \left(s_0, s_1, \ldots, s_t\right)$. The probability of state s^t as of period 0 is denoted $\tilde{\phi}\left(s^t\right)$. Commodities and prices are functions of the state s^t.

2.3.3 Consumers

The preferences of a representative consumer in the home country are represented by the expected discounted present value of utility from consumption and leisure,

$$\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\tilde{\phi}(s^t)U\left(C(s^t), 1 - N(s^t)\right) \tag{2.2}$$

The consumer faces the following budget constraint in every state s^t:

$$C(s^t) + \sum_{s_{t+1}} Q(s^t, s_{t+1}) B(s^t, s_{t+1}) \leq w(s^t) N(s^t) + B(s^t) + \Pi(s^t) + T(s^t) \tag{2.3}$$

where C denotes consumption and N is the fraction of time spent working. $Q(s^t, s_{t+1})$ is the price, in units of home country output at state s^t, of an internationally traded claim to a unit of home country output in state (s^t, s_{t+1}) and B is the quantity of these claims purchased. The wage rate, in units of domestic output, is w, and the aggregate profits Π of plants are rebated equally to all consumers. T is tariff duty collected on total imports, also rebated equally to all consumers.

Consumers have access to complete asset markets, as evident by the dependence of Q and B on the future event s_{t+1}. The consumer's ownership of the plants is modeled as passive, in that they take the profit rebate Π as given. Below, the plants' problems are specified so that their operating, entry, and technology choices are the same as those the consumer would choose for them.

The consumer's problem is to choose $C(s^t)$, $N(s^t)$ and $B(s^t, s_{t+1})$ to maximize (2.2) subject to (2.3). The first order conditions of this problem include

$$\frac{U_2(s^t)}{U_1(s^t)} = w(s^t)$$

$$Q(s^t, s_{t+1}) = \beta \phi(s^{t+1} | s^t) \frac{U_1(s^{t+1})}{U_1(s^t)} \tag{2.4}$$

where $U_j(s^t)$ is the partial derivative of U with respect to its j'th argument.

Consumers in the foreign country have the following utility function:

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \tilde{\phi}(s^t) U\left(C^*(s^t), 1 - N^*(s^t)\right)$$

and face the the budget constraint:

$$C^*(s^t) + \sum_{s_{t+1}} Q(s^t, s_{t+1}) \frac{B^*(s^t, s_{t+1})}{p(s^t)} \leq w^*(s^t) N^*(s^t) + \frac{B^*(s^t)}{p(s^t)} + \Pi^*(s^t) + T^*(s^t)$$

Here, the foreign budget constraint is written in units of foreign country output, and $p(s^t)$ is the price of foreign goods in units of home-country goods. The first order conditions for the foreign consumer's problem are:

$$\frac{U_2^*(s^t)}{U_1^*(s^t)} = w^*(s^t)$$

and

$$Q(s^t, s_{t+1}) = \beta \phi(s^{t+1}|s^t) \frac{U_1^*(s^{t+1})}{U_1^*(s^t)} \frac{p(s^t)}{p(s^{t+1})}$$

2.3.4 Plants

Plants in the economy face two types of decisions: those made at the time of establishment, and those made each period thereafter. I start with the decisions made by existing plants each period. The plant's dynamic decision at the time of establishment then anticipates the profits generated each period by the static decisions each period.

At any state s^t, a plant is distinguished by its efficiency z and its technology, importing or not. In particular, the age of a plant, reflecting the date at which it entered the economy, is irrelevant for describing its current production possibilities and decision problem, so I do not distinguish existing plants by age.

Plants operate each period under perfect competition, with decreasing returns to scale technologies. They are subject to country-specific aggregate shocks to efficiency each period, denoted $A(s^t)$ in the home country and $A^*(s^t)$ in the foreign country. These shocks are the only exogenous source of uncertainty in the economy.

Non-importing plants

The technology used by a non-importing plant with efficiency z at state s^t combines labor n and intermediate inputs d to produce output y according to:

$$y = A(s^t)z^{1-\alpha-\theta}d^{\alpha}n^{\theta}$$

where $\alpha + \theta < 1$.

The plant's static profit from operating is denoted $\pi_d(z, s^t)$, and is given by the following maximization problem:

$$\pi_d(z, s^t) = \max_{n,d\geq 0} A(s^t)z^{1-\alpha-\theta}d^{\alpha}n^{\theta} - d - w(s^t)n$$

The plant takes as given the prices of inputs in units of its output: the wage w and the price for intermediate inputs, equal to 1.

The decreasing-returns technology yields an optimal scale of production for each plant, which depends on its idiosyncratic efficiency z, and on the aggregate state s^t (through dependence on both $A(s^t)$ and the wage $w(s^t)$).

The plant's optimal input and output decisions are summarized by

$$\begin{aligned} y_d(z, s^t) &= h_d(s^t)^{1/(1-\alpha-\theta)}z \\ n_d(z, s^t) &= \frac{\theta}{w(s^t)}y_d(z, s^t) \\ d_d(z, s^t) &= \alpha y_d(z, s^t) \end{aligned} \tag{2.5}$$

where

$$h_d(s^t) = A(s^t)\alpha^{\alpha}\theta^{\theta}w(s^t)^{-\theta} \tag{2.6}$$

Plant input and output decisions are homogeneous in z. That is, for $\psi > 0$, if $z_1 = \psi z_2$, then

$$y_d(z_1, s^t) = \psi y_d(z_2, s^t)$$

and similarly for the input demands n_d and d_d.

This property of plant decisions is exploited in characterizing the model's aggregate properties below.

Maximized profits are given by

$$\pi_d(z, s^t) = (1 - \alpha - \theta) y_d(z, s^t)$$

Importing plants

An importing plant with efficiency z at state s^t produces according to:

$$y = A(s^t) z^{1-\alpha-\theta} (\gamma d^{\omega} m^{1-\omega})^{\alpha} n^{\theta}$$

Here $n, d, m,$ and y denote labor, domestic and imported intermediates, and output, respectively.

Importing plants combine intermediate inputs of domestic and imported goods to create a composite intermediate input, defined as $\gamma d^{\omega} m^{1-\omega}$, that is combined with labor. The parameter ω reflects the relative importance of domestic goods; if it is greater than $\frac{1}{2}$, then there is a technological bias within the plant towards intermediate inputs of the domestically produced good.

The parameter γ measures the efficiency advantage of the importing technology relative to the non-importing technology, discussed further in the next subsection. An efficiency advantage associated with using imported and domestic intermediate goods relative to using domestic intermediate goods alone is related to feature of "increasing returns to specialization" in the models of Ethier (1982) and Romer (1987). In these papers, production technologies are defined so that using a larger number of inputs yields higher output than using fewer inputs, in the same total quantity. Increasing returns to specialization is captured here by the parameter γ, which is calibrated in the quantitative experiments to match statistics in cross-section plant data.[10]

The profit maximization problem of an importing plant is:

$$\pi_m(z,s^t) = \max_{n,d,m\geq 0} A(s^t)z^{1-\alpha-\theta}(\gamma d^\omega m^{1-\omega})^\alpha n^\theta - d - p(s^t)(1+\tau)m - w(s^t)n$$

where $p(s^t)$ is the price of foreign country goods in units of home country goods, and τ is the *ad valorem* tariff rate. These are both taken as given by the plant, in addition to the wage $w(s^t)$.

The optimal decisions are:

$$\begin{aligned} y_m(z,s^t) &= h_m(s^t)^{1/(1-\alpha-\theta)}z \\ n_m(z,s^t) &= \frac{\theta}{w(s^t)}y_m(z,s^t) \\ d_m(z,s^t) &= \alpha\omega y_m(z,s^t) \\ m(z,s^t) &= \frac{\alpha(1-\omega)}{p(s^t)(1+\tau)}y_m(z,s^t) \end{aligned} \tag{2.7}$$

where

$$h_m(s^t) = A(s^t)\left(\gamma\alpha\omega^\omega\left(\frac{1-\omega}{p(s^t)(1+\tau)}\right)^{1-\omega}\right)^\alpha\left(\frac{\theta}{w(s^t)}\right)^\theta \tag{2.8}$$

Maximized profit for an importing plant is

$$\pi_m(z,s^t) = (1-\alpha-\theta)y_m(z,s^t)$$

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[10] In Ethier (1982) and Romer (1987), the gains from a higher number of inputs depends on substitutability between the inputs. Here, however, the inputs are assumed to be complementary in the plant's technology. Koren and Tenreyro (2005) provide an example of a production technology that yields disproportionally higher output from a larger number inputs (i.e., displays increasing returns to specialization) when the inputs are complementary.

Difference between non-importers and importers

This section considers the differences in both *potential* production possibilities and *observed* behavior between operating the importing and non-importing technologies, for a given plant with $z = 1$. Within a plant, these differences determine the realized difference in profit between importing and not, and thus impact the dynamic choice discussed in the next section.

The non-importing and importing production functions are defined over different sets of inputs. This means they cannot be meaningfully used, by themselves, to compare production possibilities, in the sense of how much output a plant gets from a given set of inputs. An alternative is to compare the total cost of production across different levels of output, measured in units of domestic goods, given that the composition of inputs is chosen to minimize total cost when using either technology.

The total (variable) cost of producing y units of output using the non-importing technology with efficiency $z = 1$ in state s^t is:

$$\begin{aligned} c_d(y, s^t) &= \min_{d,n\geq 0} d + w(s^t)n \\ & \quad \text{subject to} \\ & \quad A(s^t)d^{\alpha}n^{\theta} \geq y \end{aligned}$$

The analogue for the importing technology is:

$$\begin{aligned} c_m\left(y, s^t\right) &= \min_{d,m,n\geq 0} d + p(s^t)(1+\tau)m + w(s^t)n \\ & \quad \text{subject to} \\ & \quad A(s^t)(\gamma d^{\omega}m^{1-\omega})^{\alpha}n^{\theta} \geq y \end{aligned}$$

When minimized, these costs as functions of y are increasing and convex, and satisfy:

$$c_m(y, s^t) = \frac{c_d(y, s^t)}{\varrho(s^t)} \tag{2.9}$$

where $\varrho(s^t) = \left(\frac{\gamma}{\omega^{-\omega}(p(s^t)(1+\tau))^{1-\omega}(1-\omega)^{\omega-1}}\right)^{\alpha/(\alpha+\theta)}$. It follows that if $\varrho(s^t) > 1$, that is, if

$$\gamma > \omega^{-\omega}(p(s^t)(1+\tau))^{1-\omega}\,(1-\omega)^{\omega-1} \tag{2.10}$$

then producing with the importing technology is more cost-efficient than producing with the non-importing technology, in the sense that any level of output can be produced at lower cost. Essentially, the inequality (2.10) states that the gain in efficiency from importing (γ), is greater than the ratio of the unit price paid for intermediate goods if importing to the unit price paid for intermediate goods if only using domestic goods - the former is given by the price index of the composite of imported and domestic goods, $\omega^{-\omega}\,(1-\omega)^{\omega-1}\,(p(s^t)(1+\tau))^{1-\omega}$, and the latter is 1.

Under perfect competition, a plant's optimal scale of production sets marginal cost equal to the price of output. Denote these optimal scales $\tilde{y}_d(s^t)$ for the non-importing technology and $\tilde{y}_m(s^t)$ for the importing technology.[11] Plants operating either technology produce the same good, so the price of the output produced using either technology is the same. Therefore, these optimal levels of output must satisfy

$$\frac{\partial c_m}{\partial y}(\tilde{y}_m(s^t), s^t) = \frac{\partial c_d}{\partial y}(\tilde{y}_d(s^t), s^t) \tag{2.11}$$

Now, (2.9) holds for all y, and thus, in particular, at the optimal scale with the importing

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[11] All plant level variables with a tilde (~) and without dependence on z denote the relevant quantity for a plant with $z = 1$.

technology, $\tilde{y}_m(s^t)$. If $\varrho(s^t) > 1$, then

$$\begin{aligned}\frac{\partial c_m}{\partial y}(\tilde{y}_m(s^t), s^t) &= \frac{1}{\varrho(s^t)}\frac{\partial c_d}{\partial y}(\tilde{y}_m(s^t), s^t) \\ &< \frac{\partial c_d}{\partial y}(\tilde{y}_m(s^t), s^t)\end{aligned} \tag{2.12}$$

Since c_d and c_m are convex, $\frac{\partial c_d}{\partial y}$ and $\frac{\partial c_m}{\partial y}$ are increasing. Thus in order for (2.11) to hold, in light of (2.12), it must be that

$$\tilde{y}_m(s^t) > \tilde{y}_d(s^t)$$

Therefore, if $\gamma > \omega^{-\omega}(p(s^t)(1+\tau))^{1-\omega}(1-\omega)^{\omega-1}$, so that $\varrho(s^t) > 1$, then any plant produces at a higher scale using the importing technology than with the non-importing technology. In addition, average costs (which are proportional to marginal costs) are equal at the optimal scale using either technology, so profit is higher using the importing technology.[12] The difference in profit from using either technology is one side of the tradeoff considered by an entering plant in choosing its technology. The other side is measured by the sunk costs of either technology incurred at entry.

Entering Plant's Problem

The timing of the decisions facing a plant within the period it enters (and one period before it starts production) is as follows. An entering plant first invests κ_e to receive an efficiency z. The efficiency z is drawn independently for each entrant from a distribution with support $[z_L, \infty)$ and probability density function g. After z is revealed, a plant may decide to shut down and incur no further costs. Alternatively, it may choose to continue

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[12] If $\gamma < \omega^{-\omega}(p(s^t)(1+\tau))^{1-\omega}(1-\omega)^{\omega-1}$, then all the inequalities are reversed, so importers have less cost-efficient production technologies, are smaller in size, and have lower maximized profit than non-importers. This would contradict one fact in the data mentioned in the introduction: importing plants are, on average, larger than non-importing plants.

with future production using either of the two technologies available; the non-importing technology comes at a cost κ_c, and the importing technology at a cost κ_m. All the sunk costs of production are paid in units of domestic output.

Each plant faces uncertainty over future profits after learning its efficiency z and choosing its production technology, due to the aggregate technology shocks $A(s^t)$ and $A^*(s^t)$. Plants are also subject to a constant exogenous probability δ of exiting the economy. The timing of events is depicted in Figure 2.2.

Entrants maximize the expected present discounted value of profits from future production, less the sunk costs associated with the entry decisions. Let $V_d\left(z, s^t\right)$ denote the expected present discounted value of future profits of a plant that enters at state s^t, to begin production at date $t+1$, using the non-importing technology, with efficiency z. That is,

$$V_d(z,s^t) = \sum_{r=t+1}^{\infty} \sum_{s^r|s^t} P(s^r,s^t)(1-\delta)^{r-t-1}\pi_d(z,s^r)$$

where summation over $s^r|s^t$ refers to summation over states with histories of the form $s^r = (s^t, s_{t+1}, s_{t+2}, \ldots, s_r)$. The static profit $\pi_d(z,s^t)$ is as defined in the static maximizations of the previous section. $P(s^r,s^t)$ denotes the price of output at state s^r in units of output at state s^t, and δ is the probability that a plant dies each period. Plant death occurs at the end of the period, after production, and entering plants cannot die before they start production.

The price at which plants value future profit, $P(s^r,s^t)$ is given by

$$P(s^r,s^t) = Q(s^t,s_{t+1})Q(s^{t+1},s_{t+2})\cdots Q(s^{r-1},s_r)$$

with the Q's defined as in the consumer's problem. Using the consumer's first order condition (2.4),

$$P(s^r,s^t) = \beta^{r-t}\phi(s^r|s^t)\frac{U_1(s^r)}{U_1(s^t)}$$

That is, plants value profits at future possible states with the consumer's marginal rate of substitution.

Similarly, define $V_m\left(z, s^t\right)$ as the expected present value of profits using the importing technology:

$$V_m(z, s^t) = \sum_{r=t+1}^{\infty} \sum_{s^r|s^t} P(s^r, s^t)(1-\delta)^{r-t-1}\pi_m(z, s^r)$$

Now, the plant's decisions at entry can be characterized as follows, working backwards from the technology decision. The expected present discounted value of a plant with efficiency z that has paid the cost of entry κ_e, and has the options to exit or continue with either technology, is

$$V(z, s^t) = \max\left\{0, -\kappa_c + V_d(z, s^t), -\kappa_m + V_m(z, s^t)\right\} \tag{2.13}$$

Exiting immediately after learning z brings no additional benefits or costs, so the value of exiting is zero.

Potential entrants do not know their efficiency z before payment of the cost κ_e. The expected present discounted value for a potential entrant is then

$$V_e(s^t) = -\kappa_e + \int_{z_L}^{\infty} V(z, s^t)g(z)dz \tag{2.14}$$

An entrant's decisions are summarized by discrete decision rules determining the choice of an entrant of efficiency z at state s^t. Let $\varepsilon_d(z, s^t)$ record the decision of entrants who continue production using the non-importing technology, and let $\varepsilon_m(z, s^t)$ be the analogue for entrants who use imports. That is,

$$\varepsilon_d(z, s^t) = \begin{cases} 1 \text{ if } V(z, s^t) = -\kappa_c + V_d(z, s^t) \\ 0 \text{ otherwise} \end{cases} \tag{2.15}$$

$$\varepsilon_m(z,s^t) = \begin{cases} 1 \text{ if } V(z,s^t) = -\kappa_m + V_m(z,s^t) \\ 0 \text{ otherwise} \end{cases} \tag{2.16}$$

Aggregate Plant Dynamics

The set of plants in the economy at any date is characterized by distributions of efficiencies across plants operating each type of technology. Denote $\mu_d(z, s^{t-1})$ as the density of plants that enter a state (s^{t-1}, s_t) using the non-importing technology, with efficiency z. Similarly, $\mu_m(z, s^{t-1})$ is for importers. The mass of plants that pay the cost of entry κ_e at state s^t is denoted $X(s^t)$.

The evolution of the plant distributions follows:[13]

$$\begin{aligned} \mu_d(z,s^t) &= (1-\delta)\mu_d(z,s^{t-1}) + X(s^t)\varepsilon_d(z,s^t)g(z) \\ \mu_m(z,s^t) &= (1-\delta)\mu_m(z,s^{t-1}) + X(s^t)\varepsilon_m(z,s^t)g(z) \end{aligned} \tag{2.17}$$

That is, the set of operating plants is determined by previously existing plants that survive into the current period, along with the decisions of new entrants. For example, the mass $X(s^t)g(z)$ of new entrants with efficiency z that choose $\varepsilon_d(z,s^t) = 1$ enter the mass $\mu_d(z,s^t)$ in a manner identical to any surviving plant in $\mu_d(z,s^{t-1})$. The dependence of the distributions μ on s^{t-1} emphasizes that the set of plants in the economy at any state s^t depends only on events prior to the current period. Current decisions of new entrants affect the set of plants operating in the next period.

Aggregate Feasibility

Feasibility in the goods markets requires that the sum of demands for final and intermediate consumption, plus total goods required for investment by new plants, equal the total output

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[13] μ_d and μ_m are not necessarily *probability* distributions, because they are not normalized by the total mass of non-importing and importing plants, respectively.

produced by all plants. Plant input demands and output supplies are defined by (2.5) and (2.7) and aggregated using the distributions defined by (2.17). The total amount of goods required for $X(s^t)$ entrants is determined by the decisions in (2.15) and (2.16).

In the home country,

$$\begin{aligned} & C(s^t) + X(s^t)\left(\kappa_e + \kappa_c \int \varepsilon_d(z,s^t)g(z)dz + \kappa_m \int \varepsilon_m(z,s^t)g(z)dz\right) \\ & + \int d_d(z,s^t)\mu_d(z,s^{t-1})dz + \int d_m(z,s^t)\mu_m(z,s^{t-1})dz + \int m^*(z,s^t)\mu_m^*(z,s^{t-1})dz \\ = & \int y_d(z,s^t)\mu_d(z,s^{t-1})dz + \int y_m(z,s^t)\mu_m(z,s^{t-1})dz \end{aligned} \tag{2.18}$$

In addition, plant demands for labor must sum to total domestic labor supply:

$$\int n_d(z,s^t)\mu_d(z,s^{t-1})dz + \int n_m(z,s^t)\mu_m(z,s^{t-1})dz = N(s^t) \tag{2.19}$$

The rebates of profits and tariff revenue in the consumer's budget constraint (2.3) are defined by

$$\begin{aligned} \Pi(s^t) = & \int \pi_d(z,s^t)\mu_d(z,s^{t-1})dz + \int \pi_m(z,s^t)\mu_m(z,s^{t-1})dz \\ & -X(s^t)\left(\kappa_e + \kappa_c \int \varepsilon_d(z,s^t)g(z)dz + \kappa_m \int \varepsilon_m(z,s^t)g(z)dz\right) \end{aligned} \tag{2.20}$$

$$T(s^t) = \tau p(s^t) \int m(z,s^t)\mu_m(z,s^{t-1})dz \tag{2.21}$$

Analogues of conditions (2.18) through (2.21) hold for the foreign country.

The international asset market clearing condition is

$$B(s^t, s_{t+1}) + B^*(s^t, s_{t+1}) = 0 \tag{2.22}$$

2.3.5 Equilibrium

An equilibrium for this economy consists of state-contingent sequences of prices, allocations of goods and labor, decisions of entering plants, and distributions over efficiency levels of existing plants that solve consumers' and plants' problems and satisfy the home country and froeign country versions of the laws of motion (2.17) and feasibility conditions (2.18) through (2.21), as well as the international asset market clearing condition (2.22). In addition, the mass of entrants $X(s^t)$ must be such that

$$V_e(s^t) \leq 0, \ = \ \text{if } X(s^t) > 0$$

with $V_e(s^t)$ defined in (2.14).

2.3.6 Characterization of Equilibrium

As presented here, an equilibrium of this economy is a complicated by two things: (1) the discrete decision rules for plant technology choices at entry ε_d and ε_m; and (2) the distributions μ as equilibrium objects. The first issue can be resolved by restricting attention to equilibrium paths that satisfy a certain monotonicity condition on the difference in profits between importers and non-importers. The second issue is resolved through an explicit aggregation of plant distributions into moments relevant for the equilibrium feasibility conditions (2.18) through (2.21). Each of these issues are discussed in turn.

Plant Entry Decisions

The decision of a plant at entry involves comparing the value of the two expected discounted infinite sums in the definitions of V_d and V_m in the plant dynamic decisions. In general, it is not straightforward to determine which of these is larger for any given plant. The expected static profit difference between importing and not, discussed above, depends on future values of the endogenous price p.

To resolve this, I restrict attention to equilibrium paths that satisfy the following condition:

$$\gamma > \omega^{-\omega}(p(s^t)(1+\tau))^{1-\omega}(1-\omega)^{\omega-1} \text{ for all } s^t$$

This is not an assumption on parameters of the economy, since it involves the equilibrium price p, the relative price of foreign to home output. Rather, I compute an equilibrium path under the conjecture that this condition always holds for a given set of parameters, and then check that it does in fact hold in equilibrium, verifying the conjecture.

The reason for imposing this condition is that analysis of the plant's technology choice at entry can then be characterized by a simple rule that depends on the current state. If $\gamma > \omega^{-\omega}(p(s^r)(1+\tau))^{1-\omega}(1-\omega)^{\omega-1}$ for all s^r following s^t, then a plant entering at s^t expects to make higher profit every period it operates if it chooses the importing technology over the non-importing technology. The difference in profit is

$$\pi_m(z,s^r) - \pi_d(z,s^r) = (1-\alpha-\theta)(h_m(s^r)^{1/(1-\alpha-\theta)} - h_d(s^r)^{1/(1-\alpha-\theta)})z$$

If $\gamma > \omega^{-\omega}(p(s^r)(1+\tau))^{1-\omega}(1-\omega)^{\omega-1}$, then, from (2.6) and (2.8), the difference in profit, $\pi_m(z,s^r) - \pi_d(z,s^r)$, is increasing in z. Under the conjecture that $\gamma > \omega^{-\omega}(p(s^r)(1+\tau))^{1-\omega}(1-\omega)^{\omega-1}$ for all s^t, the difference in the present values $V_m(z,s^t) - V_d(z,s^t)$ is also increasing in z, and therefore is high enough to cover the additional sunk cost κ_m over κ_c only if z is large enough. Similar reasoning shows that $V_d(z,s^t)$ is high enough to cover the first sunk cost κ_c only for sufficiently large z as well, though for a lower range of z than for the importing decision.

Therefore, a plant's decision at entry in state s^t is characterized by two cutoff levels of its efficiency draw, denoted $\hat{z}_d(s^t)$ and $\hat{z}_m(s^t)$, with $\hat{z}_d(s^t) < \hat{z}_m(s^t)$. If a plant draws a $z \in [\hat{z}_d(s^t), \hat{z}_m(s^t)]$, it produces with the non-importing technology; if $z > \hat{z}_m(s^t)$, the plant uses the importing technology; and if $z < \hat{z}_d(s^t)$, the plant chooses not to continue producing. These cutoff rules are depicted in Figure 2.3. Across the mass of plants entering in a given period, efficiency levels z are distributed according to the fixed density g, and potential entrants along this distribution are partitioned into importers, non-importers, and exiting plants that shut down before production.

The decision rules ε_d and ε_m in (2.15) and (2.16) are replaced by

$$\varepsilon_d(z, s^t) = \begin{cases} 1 \text{ if } z \in [\hat{z}_d(s^t), \hat{z}_m(s^t)] \\ 0 \text{ otherwise} \end{cases}$$

$$\varepsilon_m(z, s^t) = \begin{cases} 1 \text{ if } z > \hat{z}_m(s^t) \\ 0 \text{ otherwise} \end{cases}$$

Therefore, an equilibrium of this economy displays two selection effects: only relatively efficient plants (those with $z \geq \hat{z}_d(s^t)$) continue beyond entry. Furthermore, only the most inherently efficient plants, those with $z > \hat{z}_m(s^t) > \hat{z}_d(s^t)$, will be profitable enough to afford the technology that uses imported intermediate inputs. These effects of sunk costs of production and importing are similar to the selection effects in Melitz (2003), in a model with sunk costs of production and exporting.

Aggregation

The endogenous state-dependent distributions $\mu_d(z, s^{t-1})$ and $\mu_m(z, s^{t-1})$ over plant efficiency can be aggregated into moments that summarize the information necessary for determining aggregate equilibrium quantities. Because the production technologies are homogeneous in efficiency z, different plants operating the same type of technology (e.g., non-importing) with different efficiencies choose inputs and outputs that are proportional to each other. So, for example, the labor demand of a non-importing plant of efficiency z

at state s^t satisfies:

$$n_d(z,s^t) = \tilde{n}_d(s^t)z$$

where $\tilde{n}_d(s^t) = n_d(1,s^t)$ (the labor demand of a non-importing plant with $z = 1$) is a function of equilibrium prices, defined by (2.5). The aggregate feasibility condition (2.19) for labor at state s^t, can then be written

$$N(s^t) = \tilde{n}_d(s^t)Z_d(s^{t-1}) + \tilde{n}_m(s^t)Z_m(s^{t-1}) \tag{2.23}$$

where Z_d and Z_m are the the following aggregates of the distributions μ_d and μ_m.

$$\begin{aligned} Z_d(s^{t-1}) &= \int z\mu_d(z,s^{t-1})dz \\ Z_m(s^{t-1}) &= \int z\mu_m(z,s^{t-1})dz \end{aligned}$$

Using these aggregate variables in addition to the cutoff rules $\hat{z}_d(s^t)$ and $\hat{z}_m(s^t)$ for entrants, the (home) goods market clearing condition can be written[14]:

$$\begin{aligned} & C(s^t) + \tilde{d}_d(s^t)Z_d(s^{t-1}) + \tilde{d}_m(s^t)Z_m(s^{t-1}) + \tilde{m}^*(s^t)Z_m^*(s^{t-1}) \\ & +X(s^t)\left(\kappa_e + \kappa_c\int_{\hat{z}_d(s^t)}^{\infty} g(z)dz + \kappa_m\int_{\hat{z}_m(s^t)}^{\infty} g(z)dz\right) \\ = \; & \tilde{y}_d(s^t)Z_d(s^{t-1}) + \tilde{y}_m(s^t)Z_m(s^{t-1}) \end{aligned}$$

In order to replace the distributions μ in summarizing the distributions of plants in the economy with the aggregates Z, the endogenous laws of motion (2.17) must also be replaced. This is done using the plant entry cutoff rules again. The aggregated laws of motion are found by mutiplying (2.17) by z for each z, and integrating over the ranges defined by the

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[14] All variables with a tilde ($\tilde{}$) and no dependence on z are defined analogously to $\tilde{n}_d(s^t)$ above.

entry cutoff rules:

$$\begin{aligned} Z_d(s^t) &= (1-\delta)Z_d(s^{t-1}) + X(s^t)\int_{\hat{z}_d(s^t)}^{\hat{z}_m(s^t)} zg(z)dz \\ Z_m(s^t) &= (1-\delta)Z_m(s^{t-1}) + X(s^t)\int_{\hat{z}_m(s^t)}^{\infty} zg(z)dz \end{aligned} \quad (2.24)$$

As with the original distributions μ, the aggregates Z at date t depend only on events up to period $t-1$, included in s^{t-1}. The aggregates evolve through the death of plants and the decisions made by new entrants.

With the plant distributions thus aggregated, solving for the aggregate variables in an equilibrium reduces to solving an aggregated maximization problem with endogenous state variables Z_d, Z_m, Z_d^*, Z_m^*. The details are in the appendix. The aggregation of plant decisions as in (2.23) is similar to the characterization in Melitz (2003) and Ghironi and Melitz (2005). Replacing the dynamics of the distributions μ with aggregated state variables is related to the method used by Atkeson and Kehoe (1999) to solve a model with "putty-clay" capital embodying an irreversibility similar to that considered here.

2.3.7 Steady state and comparative statics

In the next section I quantitatively evaluate the model's implications for changes in a country's aggregate trade flows in response to two types of movements in the relative price of imported to domestic goods. The first type are cyclical changes in $p(s^t)$ due to exogenous fluctuations in $A(s^t)$ and $A^*(s^t)$. The second type are exogenous permanent changes in trade policy, as measured by the tariff rate τ.

In this subsection I first analyze the effects of a change in the tariff τ on a symmetric steady state of the economy: an equilibrium without fluctuations in A and A^* in which all aggregate variables are constant over time. All previously defined equilibrium variables without dependence on s^t refer to steady state values. The equilibrium value of p in a symmetric steady state is 1.

Although equilibrium aggregates are constant, there is continual turnover of plants in each country, as new entrants replace dying plants. The equilibrium plant efficiency distributions μ_d and μ_m (and efficiency aggregates Z_d and Z_m) are constant, but depend on the exogenous policy τ.

Therefore, a change in τ has three effects on aggregate trade flows, two that are static and one that is dynamic. The first static effect is on the allocation of resources (labor and intermediate inputs) across existing importing and non-importing plants in any period: a reduction in tariffs reallocates resources to importing plants. The second static effect is on the ratio of imported relative to domestic intermediate inputs used within each importing plant: when imports become cheaper, importing plants use relatively more imports. The dynamic effect is on the investment decisions of new plants: a tariff reduction causes more entering plants to pay the sunk cost of importing, and causes fewer plants to continue producing at all.

These effects can be seen in the steady state ratio of aggregate imports relative to aggregate purchases of domestic intermediate goods, which is:

$$\frac{M}{D} = \frac{\int m(z)\mu_m(z)dz}{\int d_d(z)\mu_d(z)dz + \int d_m(z)\mu_m(z)dz}$$

Using the homogeneity of plant decisions in z from (2.5) and (2.7), with the definition of the aggregates Z_d and Z_m in (2.24),

$$\begin{aligned} \frac{M}{D} &= \frac{\tilde{m}Z_m}{\tilde{d}_d Z_d + \tilde{d}_m Z_m} \\ &= \frac{\tilde{m}}{\tilde{d}_m}\left(\frac{\tilde{d}_d}{\tilde{d}_m}\frac{Z_d}{Z_m} + 1\right)^{-1} \end{aligned} \tag{2.25}$$

The three effects of a drop in tariffs can be seen in the ratios $\tilde{m}/\tilde{d}_m$, $\tilde{d}_d/\tilde{d}_m$, Z_d/Z_m.

First, at importing plants, $m(z) = \frac{1-\omega}{\omega(1+\tau)} d_m(z)$, so $\tilde{m}/\tilde{d}_m = \frac{1-\omega}{\omega(1+\tau)}$: a lower tariff rate, τ, increases the ratio of imported to domestic inputs used at importing plants.

Second, using the input demand functions in (2.5) and (2.7), the ratio $\tilde{d}_d/\tilde{d}_m$ is:

$$\frac{\tilde{d}_d}{\tilde{d}_m} = \left(\frac{\omega^{-\omega}(1+\tau)^{1-\omega}\,(1-\omega)^{\omega-1}}{\gamma} \right)^{\alpha/(1-\alpha-\theta)}$$

This is increasing in τ. Therefore, a decrease in τ causes less inputs to be allocated to non-importing plants relative to importing plants, as measured by the ratio $\tilde{d}_d/\tilde{d}_m$.

Finally, The dynamic effect of a drop in τ works on the ratio M/D through the ratio of efficiency aggregates Z_d/Z_m. Evaluating the laws of motion (2.24) at a steady state give $\delta Z_d = X \int_{\hat{z}_d}^{\hat{z}_m} zg(z)dz$ and $\delta Z_m = X \int_{\hat{z}_m}^{\infty} zg(z)dz$, so the ratio is:

$$\frac{Z_d}{Z_m} = \frac{\int_{\hat{z}_d}^{\hat{z}_m} zg(z)dz}{\int_{\hat{z}_m}^{\infty} zg(z)dz}$$

I argue that the equilibrium value of this ratio decreases with a decrease in the tariff τ.

The cutoffs $\hat{z}_d$ and $\hat{z}_m$ are defined by the solutions to the steady state versions of entering plants' dynamic decision problems. The steady state versions of an entering plant's present discounted value of profits (from not importing and importing) are:

$$V_d(z) = \frac{\beta}{1-\beta(1-\delta)} \pi_d(z)$$

$$V_m(z) = \frac{\beta}{1-\beta(1-\delta)} \pi_m(z)$$

where β is the consumer's discount factor and δ is the plant's probability of death. The cutoffs $\hat{z}_d$ and $\hat{z}_m$ solve the maximization in (2.13), and therefore satisfy:

$$\frac{\beta}{1-\beta(1-\delta)} \pi_d(\hat{z}_d) = \kappa_c$$

and

$$\frac{\beta}{1-\beta(1-\delta)}\left(\pi_m(\hat{z}_m)-\pi_d(\hat{z}_m)\right)=\kappa_m$$

A plant with the cutoff efficiency level for each decision makes zero additional profit above the cost of the decision (the continuing cost κ_c for $\hat{z}_d$ and the importing cost κ_m for $\hat{z}_m$).

A decrease in τ raises the difference $\pi_m(z)-\pi_d(z)$ for any z. Since this difference is increasing as a function of z, $\hat{z}_m$ decreases, and thus more entering plants import. In addition, the equilibrium effect on $\hat{z}_d$ will typically be that, since a higher fraction of plants import, and importers hire more labor than non-importers, the equilibrium wage w increases so that fewer potential non-importing entrants are profitable enough to continue, and $\hat{z}_d$ increases.

Therefore, the integral $\int_{\hat{z}_d}^{\hat{z}_m} zg(z)dz$ decreases, and $\int_{\hat{z}_m}^{\infty} zg(z)dz$ increases, so Z_d/Z_m decreases. The dynamic effect of a tariff reduction is to increase the aggregate ratio M/D through a reduction in the mass (and aggregate efficiency, which determines aggregate intermediate demands) of non-importing plants relative to importing plants.

In the following sections, I show that these two effects interact in different ways to determine the dynamics of trade flows in response to aggregate fluctuations and in response to trade reform. Short-run fluctuations in the relative price of imports to domestic goods cause short-run fluctuations in the import/domestic ratio mainly through the static effects within and between existing plants - changes in the ratios $\tilde{m}/\tilde{d}_m$ and $\tilde{d}_d/\tilde{d}_m$ in (2.25). Trade liberalization increases trade through both the static effects and the dynamic effect of more new plants importing - a change in Z_d/Z_m. The latter effect is larger, and occurs gradually.

2.4 Quantitative Analysis

2.4.1 Parameter Values

I choose parameter values so that the steady state of the model under a tariff rate of 10% matches several aggregate statistics as well as key facts on plant-level importing behavior. The calibration is summarized in Table ??.

A model period corresponds to one quarter of a year. The discount factor β is set to 0.99, which implies an annual real interest rate of about 4%. The utility function is

$$U(C, 1-N) = \frac{\left(C^{\varsigma}(1-N)^{\varsigma}\right)^{1-\nu}}{1-\nu}$$

The parameter ζ is set to 0.34, implying that the steady state fraction of time supplied as labor, N, is 30%. The parameter ν is set to 2, a standard value in international real business cycle models (as in, for example, Backus, Kehoe and Kydland (1995)).

I set $\delta = 0.02$ based on interpreting plants as the model economy's capital stock. An accounting measure of capital in the model would cumulate investment expenditures in new plants to form a capital stock. Investment expenditures are

$$I(s^t) \equiv X(s^t)\left(\kappa_e + \kappa_c \int_{\hat{z}_d(s^t)}^{\infty} g(z)dz + \kappa_m \int_{\hat{z}_m(s^t)}^{\infty} g(z)dz\right)$$

$X(s^t)$ represents new plants entering at date s^t, a fraction δ of which will die at the end of period $t+1$. Therefore, additions to the capital stock in the form of investment expenditures I depreciate at the rate δ.

The parameters of the plant production functions that are common between non-importing plants and importing plants are α, the share of output spent on intermediate inputs, and θ, the share of output spent on labor compensation. I set $\alpha = 0.5$ and $\theta = 0.33$, so that expenditure on intermediates is the same fraction of gross output as is value added (gross output less intermediates), and labor compensation is two-thirds of value added.

In a steady state with $p = 1$, every importing plant spends a fraction $1 - \omega$ of total intermediate expenditures on imports. In US manufacturing plant data, Kurz (2006) reports an average across importing plants of 0.20 for this fraction. Kasahara and Lapham (2006), in Chilean manufacturing plant data, find an average of 0.29. Amiti and Konings (2005) find an even higher ratio of 0.46 for importing plants in Indonesia, and Halpern, Koren and Szeidl (2005) find variation in this ratio between 0.1 and 0.5 in importing Hungarian firms. I set $\omega = 0.8$ so that this fraction equals 20% for all importing plants.

The remaining parameters affect plant heterogeneity and the differences between importing plants and non-importing plants.

The parameter γ determines the advantage of using the importing technology. Several studies have attempted to measure the implicit within-plant output gain of importing intermediate inputs, given the total volume of inputs and controlling for other aspects of plant heterogeneity. The results are mixed. Kasahara and Rodrigue (2006) suggest that this gain is between 2 and 20%. Halpern, Koren and Szeidl (2005) estimate that an increase of 0.1 in a plant's import share of intermediates has a significantly positive effect on output on the order of $1 - 2\%$. Muendler (2004), however, reports no significant effect of importing on plant output among manufacturing plants in Brazil.

These three studies all use plant-level panel data to estimate a production function relating plant output to inputs (of labor, capital, and materials), augmented with a term relating to a plant's use of imported intermediate inputs. In the appendix, I construct a production function in logs, relating output to labor, total material expenditures, and a dummy variable indicating whether a plant is importing or not, for all plants. The coefficient multiplying this variable, which corresponds to the factor estimated by Kasahara and Rodrigue (2006) is:

$$\alpha \log \left(\frac{\gamma}{\omega^{-\omega} (1+\tau)^{1-\omega} (1-\omega)^{\omega-1}} \right)$$

I choose γ so that this factor is equal to 0.05. That is, any plant can produce 5% more output, given labor and total expenditures on intermediate inputs, every period (at the steady state) if it chooses the importing technology rather than the non-importing technology.

I choose the distribution over plant efficiency draws at entry to be Pareto, with probability density

$$g(z) = k(z_L)^k z^{-k-1}$$

The lower bound z_L is a normalization, so I set it equal to 3. The values of the sunk costs of entry, κ_e and continuing production, κ_c are also normalizations in that their sizes matter only relative to the sunk cost of importing, κ_m.

The cost κ_m and the shape parameter k in the distribution determine the fraction of plants in the steady state that import, and the average size difference between importers and non-importers. I turn again to the plant-level studies for these statistics. As reported in Table 2.2, about 24% of Chilean and US manufacturing plants import intermediate inputs. In Chile, these plants are over three times the size of their non-importing counterparts, and in the US they are about twice the size of non-importers. I choose the two parameters k and κ_m so that 24% of plants import and importers, on average, are 2.3 times the size of non-importers.

When simulating business cycle fluctuations, the aggregate shocks follow AR(1) processes in logs,

$$\begin{aligned}\log A(s^{t+1}) &= \rho \log A(s^t) + \varepsilon(s_{t+1}) \\ \log A^*(s^{t+1}) &= \rho \log A^*(s^t) + \varepsilon^*(s_{t+1})\end{aligned}$$

with $\rho = 0.90$ and $[\varepsilon, \varepsilon^*]$ jointly normally distributed with mean 0, standard deviation 0.005, and cross-correlation 0.25.

2.4.2 Aggregate fluctuations

In this section, I assess the model's predictions for fluctuations in the volume and balance of trade over the business cycle, and report standard business cycle statistics. First, I measure the degree to which, at the aggregate level, a country substitutes between purchases of imported and domestic goods when their relative price changes. Aggregate quantities of imported and domestic intermediate goods used in the home country at date t, denoted M_t and D_t are:

$$M_t = \int m_t(z)\mu_{mt}(z)dz$$

$$D_t = \int d_{dt}(z)\mu_{dt}(z)dz + \int d_{mt}(z)\mu_{mt}(z)dz$$

As in Ruhl (2005), I estimate the elasticity of substitution between imports and domestic intermediate goods - that is, the *Armington elasticity* - from model-generated time series of M_t, D_t, and the price p_t. To do this, I follow empirical studies such as Reinert and Roland-Holst (1992), who estimate this elasticity in US data, and estimate the following equation by least-squares regression:[15]

$$\log\left(\frac{M_t}{D_t}\right) = -\sigma \log(p_t) + b \tag{2.26}$$

The estimate of σ gives the percentage increase in the aggregate ratio M_t/D_t predicted by a one percent decrease in the price p_t. The model's time series give an estimate of σ equal to 1.96. At the aggregate level, a one percent decrease in the price of imports leads, on average, to a 1.96 percent increase in the quantity of imported interemediate goods relative to domestic intermediate goods consumed. Ruhl (2005) finds that a broad set of empirical estimates of this elasticity are in the range of about 0.2 to 3. Therefore, the model generates aggregate substitution between imported and domestic goods in line with empirical estimates.

At the model's micro level, the plant-specific ratio of imported to domestic intermediate goods is either *zero* if a plant is not an importer, or equal to $\frac{1-\omega}{\omega p_t(1+\tau)}$, if a plant is an importer. The import/domestic ratio for each importing plant responds proportionally to price changes for each plant; that is, the plant-level elasticity of substitution is equal to one. At the aggregate level, the model displays greater fluctuations in the imported-domestic goods ratio in response to price movements through the mechanisms discussed in the comparative statics exercise. Specifically, a decline in the price of imports relative to domestic goods leads existing importing plants to import more relative to their domestic inputs, and to expand in size relative to non-importing plants. In addition, the expected persistence of a price decrease leads more of the new plants entering to become importers.

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[15] In these studies, the equation is derived from the decision problem of a consumer with CES preferences over aggregate imports and domestic goods. Maximizing utility

$$U(M_t, D_t) = (\varpi D_t^{(\sigma-1)/\sigma} + (1-\varpi)M_t^{(\sigma-1)/\sigma})^{\sigma/(\sigma-1)}$$

subject to the budget constraint

$$D_t + p_t(1+\tau)M_t \leq E$$

for any E, gives (2.26) as the first order condition for the optimal M_t/D_t ratio, with the constant b depending on ϖ and τ

Table 2.5 decomposes the model's aggregate fluctuations in imports using the decomposition performed earlier on the plant-level data, as detailed in equation (2.1). Roughly, the components of the decomposition can be matched up with pieces of the comparative statics discussion above as follows: the "within" margin corresponds to the effects of changes in $m/(d+m)$, the plant-level import ratio; the "between" margin corresponds to the effect of changes in $\tilde{d}_m/\tilde{d}_d$, the average size of importing plants relative to non-importing plants; and the "entry" margin correponds to the effect of changes in Z_m/Z_d, measuring the ratio of importing to non-importing plants in the economy. The figures in Table 2.5 show that essentially all of the cyclical fluctuations in imports is attributed to the "within" and "between" margins. When compared to the decomposition done on the Chilean plant-level data, the model correctly predicts that almost all of the aggregate fluctuations in imports is accounted for by the "within" and "between" margins, and that the within-plant adjustment accounts for more of the aggregate movements than the between-plant reallocation. The fraction of aggregate fluctuations in imports accounted for by the between-plant reallocation margin is, however, much higher in the model than in the data.

Figure 2.4 presents the dynamic responses in the aggregate ratio M_t/D_t, and the three components $\tilde{m}_t/\tilde{d}_{mt}$, $\tilde{d}_{mt}/\tilde{d}_{dt}$, and Z_{mt}/Z_{dt} following a single, one-standard-deviation shock to aggregate technology in the foreign country. The relative price of imports for the home country falls. On impact, all the growth in aggregate imports relative to domestic intermediate consumption is due to the changes in $\tilde{m}_t/\tilde{d}_{mt}$ and $\tilde{d}_{mt}/\tilde{d}_{dt}$, the static within- and between-plant effects. Over time, there is a large, persistent change in the set of importing relative to non-importing plants in the economy, as measured by Z_{mt}/Z_{dt}. This large change is reflected in the time path of aggregate imports relative to domestic intermediates, M_t/D_t. Although this growth in Z_{mt}/Z_{dt} has the potential to be very large, it does not play a larger part than changes in $\tilde{d}_{mt}/\tilde{d}_{dt}$ in accounting for more of the time-series fluctuations in M_t/D_t because the growth does not have time to fully unfold when the economy is subject to recurrent fluctuations that tend to drive the relative price p_t back to its steady state value.

Table 2.6 presents business cycle statistics for the model economy and for a variation (labeled CES in the table) in which the plant-level importing decision is not present. In this variation, the sunk cost for using the importing is the same as for not importing ($\kappa_m = \kappa_c$), so all producing plants import. However, in order to make this comparable to the original model, I replace the production technology for all plants with one that features a constant elasticity of substitution between imported and domestic intermediate goods. Plants still differ by the efficiency z drawn at entry, but any plant with efficiency z produces according to the CES techology:

$$y = A(s^t)z(\upsilon d^{(\eta-1)/\eta} + (1-\upsilon)m^{(\eta-1)/\eta})^{\eta/(\eta-1)}n^{\theta}$$

The elasticity of substitution η is set equal to the estimated elasticity σ from the original model, 1.96, and the parameters υ and the sunk investment cost of production κ_c are recalibrated so that equilibrium aggregates in the steady state are the same as in the original model. All other parameters are as in Table ??.

The statistics in Table 2.6 show that, in response to fluctuations at business cycle frequency, the model's aggregate predictions are extremely similar to one in which the technology for combining domestic and imported intermediate goods simply assumes substitability at the rate estimated in the original model. One exception is that investment is slightly more volatile and less correlated across countries in the original model than in the model with CES technology. This is because in the model with all plants importing, there is one less source of variability in investment (the sunk cost to import). The relative price p is slightly less volatile and more persistent in the original model, and the trade balance, measured as the ratio of net exports to GDP, is more volatile and more persistent, than in the CES model. These differences, however, are small. In addition, these predictions are generally very close to those of standard international real business cycle models with complete asset markets, as in, for example, Backus, Kehoe and Kydland (1995).

A final remark is that the conjecture that allowed a simple characterization of equilibrium plant entry decisions can be (approximately) verified from the model's time series. Recall that, if the model's equilibrium price of foreign country goods relative to home country goods, $p(s^t)$, satisfies the inequality

$$\gamma > \omega^{-\omega}(p(s^t)\,(1+\tau))^{1-\omega}\,(1-\omega)^{\omega-1} \tag{2.27}$$

for all s^t, then the plant decision at entry is characterized in terms of two cutoffs, $\hat{z}_d(s^t)$ and $\hat{z}_m(s^t)$, of idiosyncratic efficiency z. The value of γ required for importers to be 5% more productive than non-importers is equal to 1.8583. The term $\omega^{-\omega}(p(s^t)\,(1+\tau))^{1-\omega}\,(1-\omega)^{\omega-1}$ is equal to 1.7675 when $p(s^t)=1$, its steady state value. With these parameters, the value of p would have to reach about 1.65 for the inequality (2.27) to be reversed. With the AR(1) shocks assumed here, there is no explicit bound that can be placed on the equilibrium value of $p(s^t)$, but an argument can be made that extreme values are sufficiently improbable. The maximum of the standard deviation of the price p across 1000 simulations is 3.83%. With this volatility, the price p required to violate the inequality (2.27) is about 17 standard deviations above the steady state value of 1. For the purposes of plants' evaluation of their expected profits V_d and V_m, the probability of such an extreme deviation from the steady state price is effectively zero.[16]

2.4.3 Dynamics of trade reform

I now consider the model's dynamic response to a sudden, permanent reduction in the import tariff, from 10% to 0%, when the aggregate technology shocks are constant at their

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[16] A similar argument is used by Atkeson and Kehoe (1999). However, their argument is regarding a price with an exogenous stochastic structure, and therefore applies to properties of a known distribution.

mean values of 1.[17] In response to a one-time change in the price of imported relative to domestic intermediate goods in the form of a tariff reduction, the trade dynamics suggested in Figure 2.4 gradually develop, and there is a large increase in the volume of trade.[18]

Figure 2.5 displays the same trade variables as Figure 2.4, for the first five years following the trade liberalization. The variables are, again, the ratio of aggregate imported to domestic intermediate goods, M_t/D_t; the ratio of imported to domestic inputs used by importing plants, $\tilde{m}_t/\tilde{d}_{mt}$; the ratio of goods allocated to importing relative to non-importing plants, $\tilde{d}_{mt}/\tilde{d}_{dt}$; and the ratio of aggregate efficiency of importing plants relative to non-importing plants, Z_{mt}/Z_{dt}. These ratios display similar dynamic patterns as in Figure 2.4, except that they do not eventually revert back to the original steady state. Both the static ratio of imports to deomstic inputs used by importers, $\tilde{m}_t/\tilde{d}_{mt}$, and the allocation of goods across plants measured by $\tilde{d}_{mt}/\tilde{d}_{dt}$, adjust to their new steady state levels immediately, and this adjustment drives all of the growth in trade in the period immediately following the tariff reduction. Over time, the gradual change in the number of plants importing relative to those not importing, measured by Z_{mt}/Z_{dt}, accounts for the large, gradual growth in the ratio M_t/D_t.

Figures 2.6 and 2.7 present the dynamics of other aggregate variables along the transition following the trade reform. Figure 2.6 displays GDP and its aggregate expenditure components, consumption and investment. There is a large increase in investment, as a larger proportion of new plants invest in the importing technology. Part of this increase in investment is financed by an initial reduction in consumption. GDP also increases, so that the drop in consumption is small, and consumption begins to increase relative to the original steady state after only about one year.

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[17] I compute the equilibrium path assuming that the model reaches its new steady state 100 years after the tariff reduction. This time horizon is long enough that increasing it does not significantly affect the results.

[18] This experiment is concerned with the gradual effects of a *one-time policy change.* Some previous work on the dynamic effects of trade liberalization, including Kouparitsas (1997) and Albuquerque and Rebelo (2000), studied the timing of *gradual policy changes.*

The growth in GDP is further decomposed in Figure 2.7 into changes in aggregate labor input N_t and GDP per unit of labor input, or labor productivity. In the first few periods following trade liberalization, labor increases more than GDP, so labor productivity actually falls, and only begins to grow after about three years.

Table 2.7 presents detailed measures of the magnitude and speed of the transition following trade liberalization. The first panel shows, for the trade variables and macroeconomic aggregates depicted in Figures 2.5-2.7, growth rates across steady states, and growth rates one and ten years after the tariff reduction. Both the ratios of imports to GDP and imports to domestic intermediate goods reach about half their growth within ten years. The portion of this growth due to the static allocation of resources across importing and non-importing plants is small, and is exhausted immediately. Growth in the set of new importing plants is very large, and only about one third completed after ten years. Consumption and labor productivity initially fall and then rise in the long-run, mirrored by initial increases in labor and investment higher than their respective long-run increases.

The second part of Table 2.7 again relates to Ruhl (2005), in calculating the model's implied elasticity of substitution at three different horizons following trade liberalization. At each time $t = 1$, 10, and ∞, where ∞ denotes the new free-trade steady state, the elasticity is calculated as the percentage increase in the ratio M_t/D_t relative to the original steady state, divided by the change in the relative price, reflected in the tariff reduction. That is,

$$\sigma = \frac{\left(\frac{M_t/D_t}{M/D} - 1\right)}{\left(\frac{1}{1+\tau} - 1\right)}$$

where M/D is the original steady state ratio.

After one year, the growth in trade implies an elasticity of about 2.1, which is similar to that estimated in response to business cycle fluctuations. After 10 years, the measured elasticity is about 6, and across steady states, the implied elasticity is nearly 10.

Finally, the gradual adjustment in aggregate quantities following trade liberalization suggests that there could be significant consequences for the welfare gains from trade reform. In particular, as shown in Figures 2.6 and 2.7, the initial response of the economy features a *decrease* in consumption with an *increase* in time spent working, with only a gradual increase in consumption. The welfare consequences of this can be assessed by comparing two measures of welfare gains from the trade reform.[19] The first measure compares lifetime utility across steady states, by calculating the percentage increase in the original steady state's consumption needed to attain the level of lifetime utility at the new steady state. This is the factor λ_1 that solves:

$$U(\lambda_1 C, 1-N) = U(\bar{C}, 1-\bar{N})$$

where C and N are consumption and labor supply in the original steady state, and $\bar{C}$ and $\bar{N}$ are for the free-trade steady state. The second measure of welfare gains computes an analogous consumption-variation measure, comparing lifetime utility the initial steady state to utility over the entire transition to the new steady state. That is, the second measure is the factor λ_2 that solves:

$$U(\lambda_2 C, 1-N) = \sum_{t=0}^{\infty} \beta^t U(C_t, 1-N_t)$$

where C_t and N_t are consumption and labor supply t periods following the trade liberalization.

The final panel of Table 4 shows the two measures λ_1 and λ_2. Although consumption in Figure 3 initially declines, its subsequent growth is large enough that the present value of discounted utility along the transition is larger than in the initial steady state: the consumption variation required in the initial steady state, given by $100 \times (\lambda_2 - 1)$, is 0.28%.

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[19] These calculations are similar to those in Kouparitsas (1997).

However, this is substantially lower than the analogous measure implied by λ_1, 0.72%. The initial decline and slow growth of consumption following trade liberalization therefore have significant consequences for the welfare gains of trade policy reform.

2.5 Conclusion

This paper has constructed a model of international trade in intermediate inputs used by heterogeneous plants. The model features a technological advantage for plants that use imported goods, but plants must make a costly, irreversible decision to do so. As a result, only more inherently efficient plants choose to import their intermediates.

The model is parametrized to match several features of plant-level importing behavior. When the model is subject to short-run fluctuations driven by aggregate technology shocks, it generates low volatility of trade flows. A low degree of aggregate substitution between imports and domestic goods in the short-run is acheived through shifts in the allocation of resources within and across across importing and non-importing plants.

In response to a sudden, permanent trade liberalization, the set of plants in the economy gradually changes. A higher proportion of new plants import intermediates. Existing plants cannot change their production technologies, but gradually die out. Over a very long time horizon, imports double as a fraction of GDP in response to the one-time removal of a 10% tariff; however, along the transition path, only about half of this increase is attained within 10 years. The welfare gain calculated from the transition following trade liberalization is significantly lower than that computed from comparing steady states.

The model provides a framework for analyzing the dynamic effects of trade policy through changes in producer-level importing decisions. With irreversibility in these decisions, changes in trade policy have both static and dynamic effects on the allocation of resources across plants that import and plants that do not. These contribute to very large effects on trade flows that occur gradually over time.

The model here has focused on the plant-level decision to import, motivated by recent emprical evidence of the importance of this decision. A large body of evidence exists as well for the importance of the plant-level exporting decision, and a useful extension would be to integrate the dynamic plant-level importing decisions introduced here with the exporting decisions analyzed in much of the recent trade literature.

2.6 Appendix

2.6.1 Aggregation

For any plant-level variable $j_q(z, s^t)$, with $q = m$ or d, define the corresponding equilibrium aggregate by $J_q(s^t) = \int j_q(z, s^t)\mu_q(z, s^{t-1})dz$. Aggregating the plant decision rules in (2.5) and (2.7) shows that

$$Y_d(s^t) = A(s^t)Z_d(s^{t-1})^{1-\alpha-\theta}D_d(s^t)^{\alpha}N_d(s^t)^{\theta}$$

$$Y_m(s^t) = A(s^t)Z_m(s^{t-1})^{1-\alpha-\theta}\left(\frac{\gamma}{\omega}\right)^{\alpha}D_m(s^t)^{\alpha}N_m(s^t)^{\theta}$$

where Z_d and Z_m are defined in (2.24).

The aggregated version of the feasibility conditions can be written as follows.

Home country goods feasibility:

$$\begin{aligned}
& C(s^t) + D_d(s^t) + D_m(s^t) + (1+\tau)M^*(s^t) - T^*(s^t) \\
& +X(s^t)\left(\kappa_e + \kappa_c \int_{\hat{z}_d(s^t)}^{\infty} g(z)\,dz + \kappa_m \int_{\hat{z}_m(s^t)}^{\infty} g(z)\,dz\right) \\
= \ & A(s^t)Z_d(s^{t-1})^{1-\alpha-\theta}D_d(s^t)^{\alpha}N_d(s^t)^{\theta} + A(s^t)Z_m(s^{t-1})^{1-\alpha-\theta}\left(\frac{\gamma}{\omega}\right)^{\alpha}D_m(s^t)^{\alpha}N_m(s^t)^{\theta}
\end{aligned} \tag{2.28}$$

Foreign country goods feasibility:

$$\begin{aligned}
& C^*(s^t) + D_d^*(s^t) + D_m^*(s^t) + (1+\tau)M(s^t) - T(s^t) \\
& +X^*(s^t)\left(\kappa_e + \kappa_c \int_{\hat{z}_d^*(s^t)}^{\infty} g(z)\,dz + \kappa_m \int_{\hat{z}_m^*(s^t)}^{\infty} g(z)\,dz\right) \\
= \ & A^*(s^t)Z_d^*(s^{t-1})^{1-\alpha-\theta}D_d^*(s^t)^{\alpha}N_d^*(s^t)^{\theta} + A^*(s^t)Z_m^*(s^{t-1})^{1-\alpha-\theta}\left(\frac{\gamma}{\omega}\right)^{\alpha}D_m^*(s^t)^{\alpha}N_m^*(s^t)^{\theta}
\end{aligned} \tag{2.29}$$

Home country labor feasibility:

$$N_d(s^t) + N_m(s^t) \leq N(s^t) \tag{2.30}$$

Foreign country labor feasibility:

$$N_d^*(s^t) + N_m^*(s^t) \leq N^*(s^t) \tag{2.31}$$

The aggregated laws of motion for the state variables are as follows.

For the home country:

$$Z_d(s^t) = (1-\delta)Z_d(s^{t-1}) + X(s^t)\int_{\hat{z}_d(s^t)}^{\hat{z}_m(s^t)} zg(z)dz \tag{2.32}$$

$$Z_m(s^t) = (1-\delta)Z_m(s^{t-1}) + X(s^t)\int_{\hat{z}_m(s^t)}^{\infty} zg(z)dz \tag{2.33}$$

For the foreign country:

$$Z_d^*(s^t) = (1-\delta)Z_d^*(s^{t-1}) + X^*(s^t)\int_{\hat{z}_d^*(s^t)}^{\hat{z}_m^*(s^t)} zg(z)dz \tag{2.34}$$

$$Z_m^*(s^t) = (1-\delta)Z_m^*(s^{t-1}) + X^*(s^t)\int_{\hat{z}_m^*(s^t)}^{\infty} zg(z)dz \tag{2.35}$$

The presence of the tariff τ along with the rebates T in the feasibility conditions allows the incorporation of the distortions arising from import tariffs in the aggregated planning problem.[20] The planning problem is, *given* sequences of $T(s^t)$ and $T^*(s^t)$ and initial values of $Z_d(s^0), Z_d^*(s^0), Z_m(s^0), Z_m^*(s^0)$, to maximize an equally-weighted sum of home and foreign

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[20] This method follows Kehoe, Levine and Romer (1992).

consumers' utilities,

$$\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\tilde{\phi}(s^t)\left[U(C(s^t),N(s^t))+U(C^*(s^t),N^*(s^t))\right]$$

subject to (2.28) through (2.35) for all s^t, by choosing:

1. Consumption and labor for consumers, C, C^*, N, and N^*;
2. Allocations of inputs, $D_d, D_d^*, D_m, D_m^*, M, M^*, N_d, N_d^*, N_m$, and N_m^*;
3. Mass of new plants X and X^*;
4. Cutoffs $\hat{z}_d, \hat{z}_d^*, \hat{z}_m$, and $\hat{z}_m^*$; and
5. Future values of the state variables Z_d, Z_d^*, Z_m, and Z_m^*.

A "side condition" imposed on this problem is that the choices for M and M^* satisfy the following:

$$\begin{aligned} T(s^t) &= \tau M(s^t) \\ T^*(s^t) &= \tau M^*(s^t) \end{aligned}$$

The equivalence between this planning problem and an equilibrium of the original model is established through a comparison of the first order conditions of this problem and the equilibrium conditions from consumers' and plants' decisions in the original model.

2.6.2 Calibrating γ

Although the two production functions for importing and non-importing plants in the model are defined over different sets of inputs, a production function relating output to labor and *total expenditures* on intermediate inputs (which are in the same units for all plants) can be defined as follows. Let x_d and x_m denote total expenditures on intermediate inputs for a non-importing plant and an importing plant, respectively. For any non-importing plant,

$$x_d = d_d$$

where d_d is from the original production function. A plant with efficiency z, using intermediate inputs x and labor n produces output

$$y = z^{1-\alpha-\theta} x^{\alpha} n^{\theta}$$

For an importing plant,

$$x_m = d_m + (1+\tau)m$$

Now, for any importing plant, $m = \frac{1-\omega}{\omega(1+\tau)} d_m$. Therefore,

$$x_m = \frac{d_m}{\omega}$$

The output produced by a plant operating the importing technology with efficiency z is then

$$y = z^{1-\alpha-\theta} \left(\gamma \omega^{\omega} \left(\frac{1-\omega}{1+\tau} \right)^{1-\omega} x \right)^{\alpha} n^{\theta}$$

Across all plants, the production function is:

$$y = \begin{cases} z^{1-\alpha-\theta} x^{\alpha} n^{\theta} \text{ if a plant does not import} \\ z^{1-\alpha-\theta} \left(\frac{\gamma}{\omega^{-\omega}(1+\tau)^{1-\omega}(1-\omega)^{\omega-1}} \right)^{\alpha} x^{\alpha} n^{\theta} \text{ if it does} \end{cases}$$

Taking logs, the following production function with a dummy variable indicating importing status applies to all plants:

$$\log y = (1-\alpha-\theta)\log z + \alpha \log x + \theta \log n + \alpha \log \left(\frac{\gamma}{\omega^{-\omega}(1+\tau)^{1-\omega}(1-\omega)^{\omega-1}} \right) \chi$$

where $\chi = 1$ if the plant imports and $\chi = 0$ if not.

Therefore, the term $\alpha \log \left(\frac{\gamma}{\omega^{-\omega}(1+\tau)^{1-\omega}(1-\omega)^{\omega-1}} \right)$ measures the percentage increase in a given plant's output if it imports relative to if it does not. This is the analogue of the statistic estimated in Kasahara and Rodrigue (2005), and is related to the one measured in Halpern, Koren and Szeidl (2005) and Muendler (2004).

Table 2.1: Imported Intermediate Inputs in World Trade

Country	$\frac{\text{Intermediates}}{\text{Merchandise Imports}}$	Year
Australia	0.35	1994-5
Brazil	0.52	1996
Canada	0.39	1997
China	0.62	1997
Czech Republic	0.49	1995
Denmark	0.35	1997
Finland	0.56	1995
France	0.47	1995
Germany	0.43	1995
Greece	0.27	1994
Hungary	0.57	1998
Italy	0.51	1992
Japan	0.50	1995
Korea	0.63	1995
Netherlands	0.34	1995
Norway	0.32	1997
Poland	0.49	1995
Spain	0.52	1995
United Kingdom	0.37	1998
United States	0.34	1997

Source: OECD Input-Output Tables. Ratio reported is the fraction of manufacturing, mining, and agricultural imports used as intermediate inputs by manufacturing, mining, and agricultural industries.

Table 2.2: Cross-section Plant Characteristics

	Importers (%)	Size Ratio
Chile, 1979-86	24.1	3.4
US, 1992	23.8	2.3

Source: Chile, *INE* Survey; US, Kurz (2006). Size ratio is average employment of importing plants divided by average employment of non-importing plants.

Table 2.3: Decomposition of Aggregate Imports, Chile 1979-86

	% Change	% of Total				
Time period	Total	Within	Between	Cross	Switch	Entry
1 year†	−18	79	26	−10	3	2
7 years	−77	74	42	−30	5	10

Data from Chile's *INE* Survey. See text and equation (2.1) for column definitions.

†Average across 1-year changes.

Table 2.4: Calibration

Parameter	Role	Value	Chosen to Match
β	discount factor	0.99	annual $r = 0.04$
ζ	share on c in utility	0.34	$N = 0.3$
ν	intertemporal elasticity	2.00	standard value
α	intermediates / gross output	0.50	$\frac{INT}{GDP} = 1.00$
θ	wN / gross output	0.33	$\frac{wN}{GDP} = 0.66$
γ	advantage of importing	1.86	*see text*
ω	home bias	0.80	$\frac{m}{d_m} = 0.20$
δ	plant death rate	0.02	capital depreciation
z_L	distribution lower bound	3.00	normalization
κ_e	entry cost	0.05	normalization
κ_c	non-importing technology cost	0.25	normalization
κ_m	importing technology cost	0.38	*see text*
k	distribution shape parameter	3.75	*see text*
ρ	autocorrelation of shocks	0.90	$\text{corr}(TFP_t, TFP_{t-1}) = 0.90$
σ_ε	std of shocks	0.005	$\sigma_{TFP} = 0.01$
$\text{corr}(\varepsilon, \varepsilon^*)$	correlation of shocks	0.25	$\text{corr}(TFP, TFP^*) = 0.25$

Table 2.5: Decomposition of Aggregate Imports, Model and Chilean Plant Data

	% of Total				
	Within	Between	Cross	Switch	Entry
Model	57	49	0	0	−6
Data	79	26	−10	3	2

Model: Medians of 1000 120-quarter simulations, annualized.

Data: Table 2.3.

See text and equation (2.1) for column definitions.

Table 2.6: Model Business Cycle Statistics

	$\text{std}(x)^\dagger$		$\text{corr}(x, GDP)$		$\text{corr}(x, x^*)$		$\text{corr}(x_t, x_{t-1})$	
Variable, x	Model	CES	Model	CES	Model	CES	Model	CES
GDP	1.88	1.88	1.00	1.00	0.23	0.20	0.67	0.67
Consumption	0.27	0.28	0.95	0.95	0.41	0.39	0.73	0.72
Investment	3.76	3.68	0.99	0.99	0.07	0.12	0.66	0.66
Labor	0.52	0.52	0.99	0.99	0.19	0.20	0.66	0.66
p	0.24	0.25	0.49	0.49			0.83	0.82
Net Exports / GDP	0.07	0.06	−0.50	−0.39			0.73	0.77

Means of statistics over 1000 simulations of 120 quarters each. CES variant of the model is described in the text. All variables except net exports are logged and Hodrick-Prescott filtered. †For GDP, percent standard deviation; for all other variables, ratio of standard deviation to that of GDP.

Table 2.7: Dynamics of Trade Liberalization

	Percent growth rate		
	steady states	after 1 year	after 10 years
Imports / GDP	82.89	18.67	52.45
M / D	93.03	18.54	56.70
$\tilde{m}$ / $\tilde{d}_m$	10.01	10.01	10.01
$\tilde{d}_m$ / $\tilde{d}_d$	5.76	5.76	5.76
Z_m / Z_d	195.45	6.69	78.75
GDP	1.53	0.92	1.35
Consumption	1.31	0.02	0.92
Investment	2.31	4.02	2.83
Labor (N)	0.69	1.16	0.83
GDP / N	0.84	−0.24	0.51

Implied elasticity of substitution		
steady states	after 1 year	after 10 years
9.73	2.10	6.05

Percent welfare gain	
$100(\lambda_1 - 1)$	$100(\lambda_2 - 1)$
0.72	0.28

Figure 2.1: Mexico: Imports from US relative to US GDP and Average Tariff on US Goods

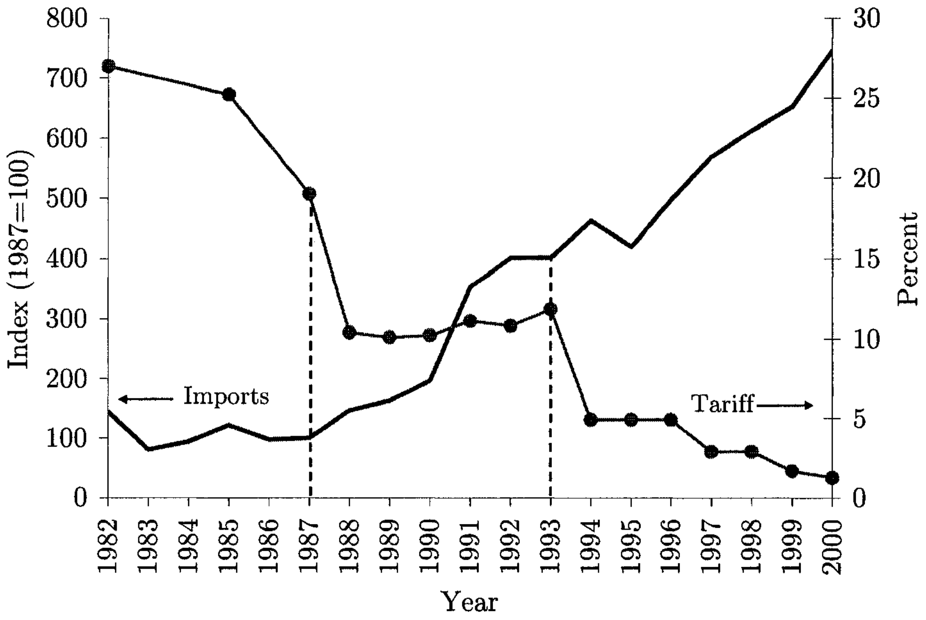


Figure 2.2: Dynamic decision of a plant entering at state s^t

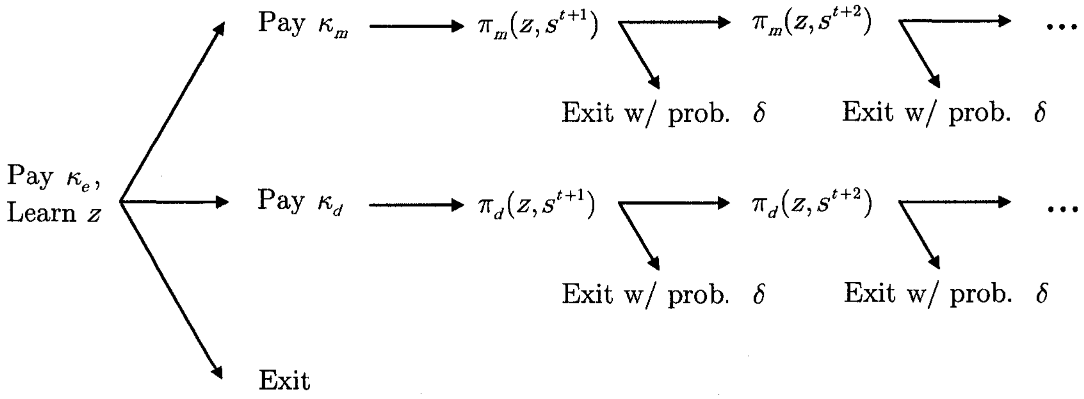


Figure 2.3: Technology choice cutoffs across entering plants

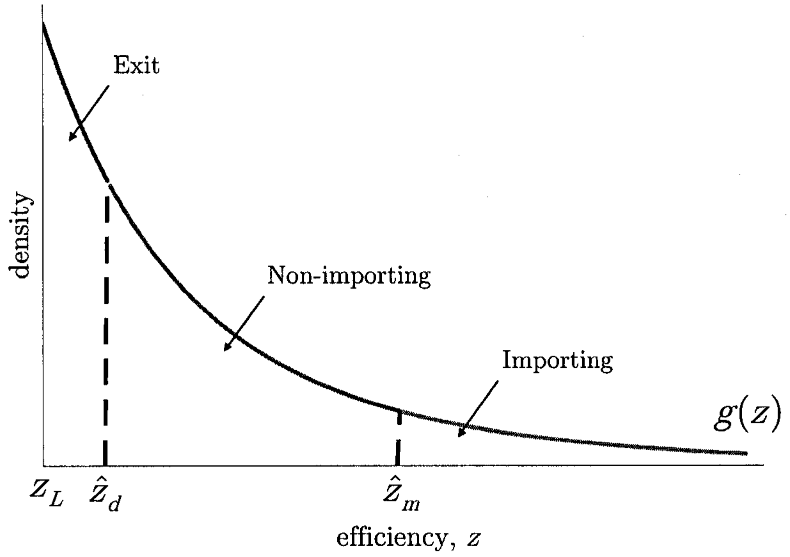


Figure 2.4: Dynamic responses to a one-standard-deviation shock to A^*

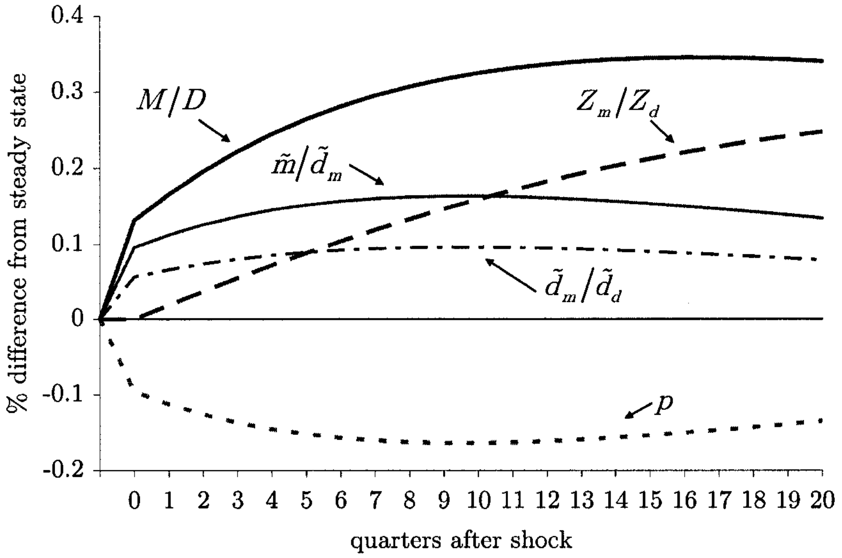


Figure 2.5: Dynamic responses following trade reform: Trade variables

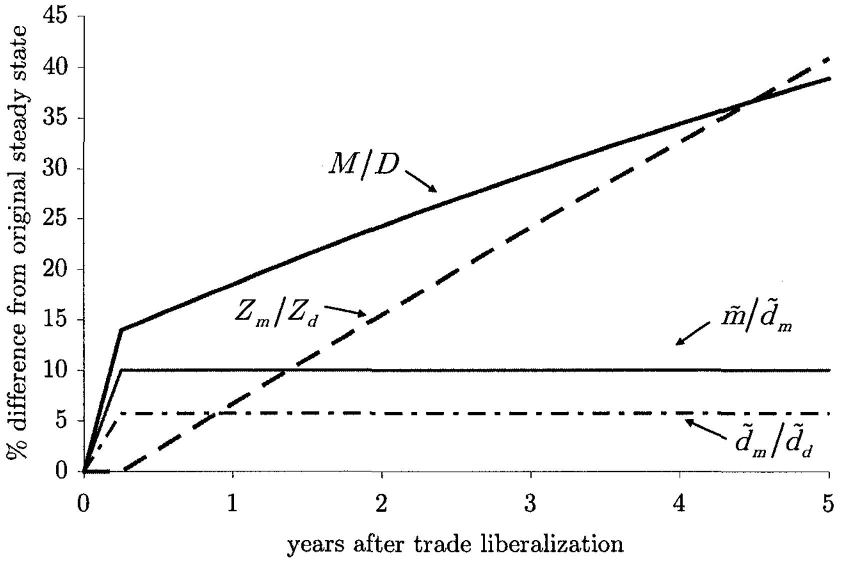


Figure 2.6: Dynamic responses following trade reform: GDP, Consumption, and Investment

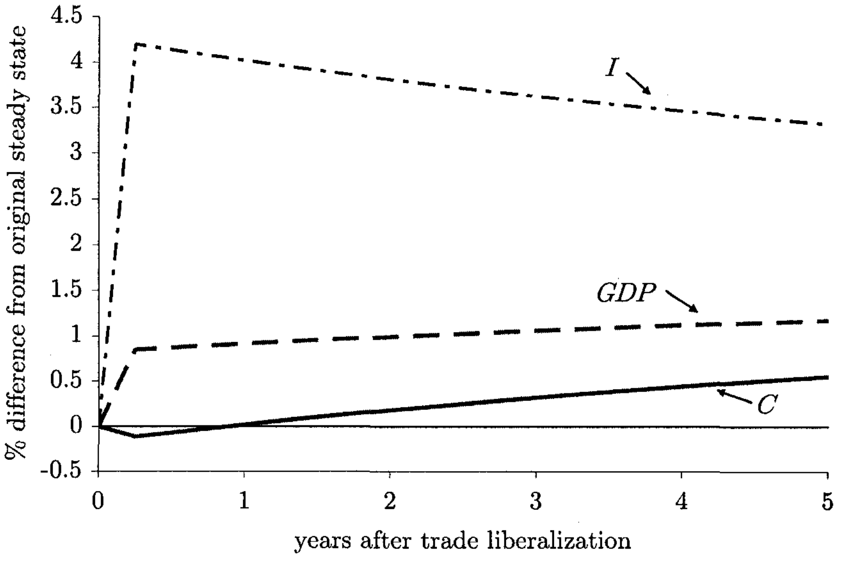
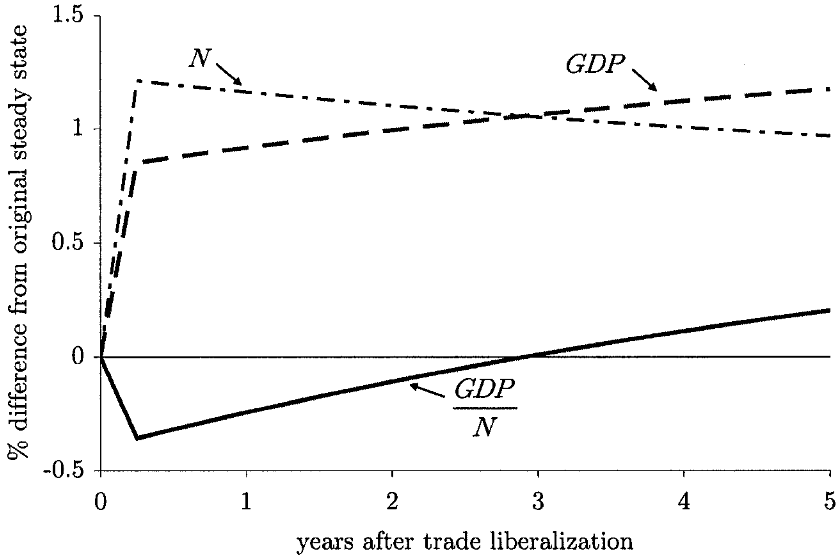


Figure 2.7: Dynamic responses following trade reform: GDP and Labor



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Chapter 3

Default and the Term Structure in Sovereign Bonds

3.1 Introduction

During the last decade emerging economies have increased the set of foreign bonds they issue in international financial markets, moving more towards longer maturity debt. Broner, Lorenzoni and Schmukler (2005) document that government foreign debt in emerging economies is mostly of long maturity, with relatively small amounts of debt issued at maturities of 3 years or less. In addition the term structure of emerging markets foreign debt presents some salient features. First the spread curve is on average upward sloping, with long spreads being higher than short spreads. Second, around crises times, the spread curve inverts with short spreads being higher than long spreads. Lastly the maturity of debt issuances correlates with emerging markets domestic conditions. In particular, emerging markets issue long bonds mostly in tranquil times and issue short debt during crises. We document these facts more detail for a set of foreign bonds issued by the government of Brazil.

This paper constructs a dynamic model of borrowing and default to study the term structure of sovereign bonds. In the model a sovereign borrower can issue long and short maturity bonds and can default on them at any point in time. The spreads the borrower pays on these bonds reflect his default probabilities because lenders are compensated for possible default events and for risk premia. Default probabilities and interest rates both short term and long term are endogenous to the borrower's default incentives. The model generates a spread curve that is upward sloping in tranquil times with long spreads being higher than short spreads on average. The reason is that if default events are likely in the future but not in the near term, only the long spread will be adjusted for this. On the other hand if default is a likely event only in the short term, but not in the long term then the annualized rates for short bonds will be higher than those for long bonds. Long bonds are safer for lenders than short bonds in present value terms, because if the economy avoids the stressed period, it may repay its debt obligations in all future states.

The model also generates that long bonds are issued primarily on tranquil times, and short debt is used more heavily during crisis as in emerging markets. In the model long debt provides a good hedge against future bad shocks because the effective cost for such borrowing is lower exactly in times of high interest rates. In fact by simultaneously borrowing long term and saving short term the borrower can relax borrowing constraints in future bad times quite cheaply. Thus the borrower prefers in tranquil times long bonds because of the additional benefits of completing markets.

The model is calibrated to Brazil and can generate various facts of the Brazilian bond market. First the model matches the volatility of long and short spreads with long bonds spreads being less volatile on average than short spreads. The model also generates that prior to a default, the spread curve is inverted with short spreads being larger than long spreads. In addition in the model the economy primarily borrows long term in times of good shocks, and borrows short term in times of bad output shocks as in the data.

The optimal maturity of debt in emerging countries is a topic of special interest because of the general view that countries could alleviate their vulnerability to crises by choosing the appropriate maturity structure. In particular, by lengthening the maturity of debt and reducing the dependence on short term debt, countries could manage better external shocks and sudden stops. For example Cole and Kehoe (2000) argue that the 1994 Mexican crisis could had been managed better if not for the government dependence on Tesobonos, that were very short maturity instruments. This paper contributes to this debate by analyzing default decisions and borrowing incentives in a dynamic model of equilibrium default where the prices of debt reflect the timing of default.

The paper builds on the work by Aguiar and Gopinath (2005) and Arellano (2005) who model equilibrium default with incomplete markets as in the seminal paper on sovereign debt by Eaton and Gersovitz (1981). This paper extends such framework to incorporate assets of multiple maturities to study more broadly the spread curve in sovereign bond markets and its ability to account for the term structure regularities.

Broner, Lorenzoni, and Schumukler (2005) also study the optimal maturity structure and debt issuances but focus primarily on the lender's side. They argue that countries borrow in short bonds because they are cheaper in that they do not have to include compensation for varying short rate when lenders are risk averse and face liquidity needs. In this framework the borrower also chooses the optimal maturity structure based on the costs of both assets, however the differential cost is due to the timing of defaults.

3.2 Brazil Bond Data

We examine data on 46 government bond issues by Brazil in international markets. The source for these data is Bloomberg. The bonds' maturities when issued vary between 2 and 30 years, and their issue dates range between December 1988 and March 2000. Most of the debt consists of long bonds: of the total dollar value of these issues, 93% is of maturity longer than 5 years when issued. Table 3.1 highlights that the maturity of debt is longer in tranquil times than in Brazil's crises in 1998 and 1999. The issue-amount-weighted average maturity is over 18 years during a period of high debt issue in 1996-1997, but less than 7 years during 1998-2000.

We use end-of-week price quotes to compute yields and spreads over risk-free rates for corresponding maturities.[1] The price data approximately cover the period November 1996 to March 2006. At different dates within this range, potentially different sets of maturities are available, so we estimate the term structure of the spreads at every date using the method of Nelson and Siegel (1987). The appendix describes the procedure in more detail. Figure 3.1 shows estimated spread curves on two days specifically chosen to illustrate normal and crisis periods.[2] During normal times, spreads of all maturities are lower and the spread curve is upward sloping, and during crises, spreads are higher and the spread curve inverts.

Figure 3.2 shows time series for short and long spreads.[3] The time series maintain the pattern of the previous figure: spreads are normally upward sloping across maturity, but flatten or even invert during the crises of 1998-99 and 2002. Spreads on all maturities increase during crises, but short spreads increase relatively more than long spreads. As a result, as shown in Table 3.2, the spread curve is on average upward sloping, and spreads of short maturities are more volatile than those of long maturities.

Finally, with spread curves calculated, we can examine both movements in yield spreads and maturities of bond issues during crises. We find that for Brazil average maturity of bond issues co-moves negatively with short spreads. As Table 3.1 shows, between 1997-1999, the Brazilian government issued more shorter maturity bonds than between 1996-1997. The average 2 year spread was 7.3% for 1996-1997 and 10.2% for 1997-1999. Thus periods of low spreads were associated with longer maturity issuances. Similar patterns can be seen

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[1] The spreads are calculated as the difference between the Brazilian yield and the yield of a risk free bond of the same maturity issued by US treasury if the Brazilian bond is in dollars or by the European Central Bank if the Brazilian bond is in Euros.

[2] The solid lines are estimated spread curves and the dots are the sample spreads from the data we use in calculating the curves.

[3] The gaps during the 1998-99 crisis are due to an absence of quoted prices for different maturities.

in Figure 3.3 where we plot the maturity and dollar amount issuance of all the bonds we have data for.

3.3 Model

The model consists of a small open economy that receives every period a stochastic stream of output y_t of a tradable good that follows a Markov process. The borrower who is the representative agent of the economy trades with lenders bonds of short and long maturity that pay an uncontingent amount. Financial contracts are unenforceable in that the borrower can default on his debt whenever he wants to. In case of failure to repay in full all its debt obligations, the economy incurs costs that consist on lack of access to international financial markets and direct output costs.

In the model two types of bonds are issued by the economy. First, b_{t-1} denotes one-period zero coupon debt outstanding at time t. This bond is a promise to pay one unit of consumption in all states. Second, b^2_{t-2} denotes the two-period zero coupon debt outstanding at t.

The stand-in agent has standard preferences

$$E\sum_{t=0}^{\infty}\beta^t u(c_t)$$

The agent's budget constraint conditional on not defaulting is standard. Its purchases of the single consumption good in the spot markets is constrained by its endowment less payments of the one-period and two-period zero coupon bonds, plus the issues of new zero coupon debt b_t at price q^1_t and two-period bonds b^2_t at a price of q^2_t:

$$c_t - q^1_t b_t - q^2_t b^2_t = y_t - b_{t-1} - b^{2,}_{t-2}$$

In particular, in every period the agent chooses its debt holdings from a menu of contracts where prices q_t^1 and q_t^2 for are quoted for each pair (b_t, b_t^2).

In case of default, we assume that current debts are erased from the budget constraint of the agent and that it cannot borrow or save such that consumption equals output. In addition, the country incurs output costs.

$$c_t = y_t^{def}$$

where $y_t^{def} = h(y) \leq y$.

3.3.1 Lenders

Lenders in this economy are competitive and discount time at rate $\delta < 1$. Lenders receive an exogenous stochastic stream of consumption c_L that follows a Markov process and their lifetime value is given by: $E\sum_{t=0}^{\infty}\delta^t u_L(c_{L,t})$. They behave passively and are willing to hold the small open economy defaultable bonds, as long as they are compensated for the expected loss in case of default and for risk premia. When $u_L(c_{L,t}) = c_{L,t}$ lenders are risk neutral and the only compensation for lenders is for the loss of principal in the event of default. While when lenders are risk averse, they are also compensated for variations in default probabilities and variations in the short term rate. Effectively, lenders in the model simply provide a pricing kernel that is used to price the small open economy defaultable debt. The focus here is on the interaction of default events and risk premia on the small open economy debt contracts, thus we model directly lender's consumption as a stochastic process.

The implicit assumption is that the payoff from operations with the economy is small enough such that it doesn't affect lenders' aggregate consumption. However if default events are correlated with investors consumption, the price of loans will be affected. In particular, lenders will require a premium over and above the risk of default to hold the economy's asset if default events are likely to happen in low consumption times to compensate for risk.

3.3.2 Recursive Problem

For a given schedules for debt, the recursive problem of the borrower can be represented by the following dynamic programming problem.

Let $x_t = \{y_t, c_{t,L}\}$ be the exogenous state of the model which consists of the realization of the lender's consumption and the economy's output. We denote by $x^t = (x_0, ...x_t)$ the history of events up to and including period t. Given that both shocks are Markov we denote $f(x', x)$ the joint conditional density for the two stochastic variables of the model. Let's also define the endogenous states of the economy by the total cash on hand: $b^1_{t-1} + b^2_{t-2}$ which consists of previous period outstanding one-period debt and outstanding long term debt purchased two periods before, and by the outstanding long debt purchased the previous period that is due the following period b^2_{t-1}. The states for the model then include the endogenous and exogenous states $s \equiv (b, b^2, x) = (b^1_{t-1} + b^2_{t-2}, b^2_{t-1}, x_t)$.

Given that initial states are s, the value of the option to default is given by

$$v^0(b, b^2, x) = \max\left\{v^c(b, b^2, x), v^d(x)\right\}$$

where $v^c(b, b^2, x)$ is the value associated with not defaulting and staying in the contract and $v^d(x)$ is the value associated with default. Given that default costs are incurred whenever the borrower fails to repay in full its obligations, the model will only generate complete default on all outstanding debt short and long term.

When the borrower defaults, the economy is in temporary financial autarky; θ is the probability that it will regain access to international credit markets. If the borrower defaults, output falls and equals consumption. The value of default is given by the following:

$$v^d(x) = u(y^{def}) + \beta \int_{x'} \left[\theta v^o(0, 0, x') + (1-\theta) v^d(x')\right] f(x', x) dx' \tag{3.1}$$

We are taking a reduced form specification to model both costs of default that seem empirically relevant: exclusion from financial markets and direct costs in output.

When the agent chooses to remain in the credit relation, the value conditional on not defaulting is the following:

$$v^c(b, b^2, x) = \max_{\{b', b^{2\prime}\}} \left(u(c) + \beta \int_{x'} v^0(b', b^{2\prime}, x') f(x', x) dx \right)$$

subject to the law of motion for short term debt:

$$b' = b^2 + \Delta b'$$

and subject to the budget constraint:

$$c - q^1 \Delta b' - q^2 b^{2\prime} = y - b$$

The borrower decides on optimal contracts b' and b^2 to maximize utility. The borrower understands that each contract $\{b',\ b^{2\prime}\}$ comes with specific prices $\{q^1,\ q^2\}$. The decision to remain in the credit contract and not default is a period-by-period decision so that the expected value from next period forward incorporates the fact that the agent could choose to default in the future.

The default policy can be characterized by default sets and repayment sets. Let $A(b, b^2)$ be the set of $x's$ for which repayment is optimal when debt positions for short and long term are (b, b^2), such that*:*

$$A(b, b^2) = \left\{ x \in X : v^c(b, b^2, x) > v^d(x) \right\},$$

and let $D(B) = \widetilde{A}(B)$ be the set of $x's$ for which default is optimal for debt positions (b, b^2), such that

$$D(b,b^2) = \left\{x \in X : v^c(b,b^2,x) \leq v^d(x)\right\}. \tag{3.2}$$

3.3.3 Bond Prices

The price schedules are functions of the agent's endogenous states next period which determine the default decision and debt policy, and the current stochastic variables which determine the likelihood of the stochastic shock tomorrow: $\{q_t^1(b_t + b_{t-1}^2, b_t^2, x_t), q_t^2(b_t + b_{t-1}^2, b_t^2, x_t)\}$.

The price for the one-period economy's loan is then given by the lender's pricing kernel:

$$q^1(b', b^{2\prime}, x) = \delta \int_{A(b', b^{2\prime})} \frac{u_L'(c_L')}{u_L'(c_L)} f(x', x) dx'$$

For every pair $(b', b^{2\prime})$ the lender offers a price that compensates for the possible default event where the payoff will be zero, and for bearing the risk of default if the event correlates with their consumption. Specifically if default events are likely when the lender's consumption is low, the price on these loans will be lower than the default adjusted payoff. And if default events are likely when the lender's consumption is high, the price will be higher than the default adjusted payoff.

Given that default occurs for all outstanding debt simultaneously, the price for the two-period bond incorporates default probabilities for the next period and for two periods ahead which is when the bond is due. The equilibrium price for the two-period bond also needs to forecast future debt holding, because the probability of default in the future depends on all debt holdings until the bond is due.

Let's first define a transition law such that:

$$Q(b', b^{2\prime}; s) = \left\{ \begin{array}{l} 1 \text{ if } b'(b, b^2, x) = b' \text{ and } b^{2\prime}(b, b^2, x) = b^{2\prime} \\ 0 \text{ elsewhere} \end{array} \right\}$$

The two-period bond price is the present value of one unit of consumption discounted by the possible loss from default in the following two periods and by the compensation for risk if default probabilities correlate with the lender's marginal rate of substitution.

$q^2(b', b^{2\prime}, x) =$

$$\delta^2 \left[\int_{A(b',b'^2)} \frac{u'(c'_L)}{u'(c_L)} f(x',x) \left[\int_{A(b'',b^{2''})\times B} \frac{u'(c''_L)}{u'(c'_L)} Q(b'',b^{2''};s') f(x'',x') d(b'',b^{2''},x'') \right] dx' \right]$$

Note that if default sets are empty in the following two periods, the price of the two-period bonds collapses to the standard default free long discount price $q^2 = \delta^2 E\left[\frac{u'(c''_L)}{u'(c_L)}\right]$.

Under risk neutrality, marginal utility equals one, and thus the above formulas take into account only default risk and not risk premia.

3.3.4 Equilibrium

We now define the equilibrium:

Definition. *The recursive equilibrium for this economy is defined as a set of policy functions for (i) consumption* $c(s)$*, short term debt holdings* $b'(s)$*, long term debt holdings* $b^{2\prime}(s)$*, repayment sets* $A(b, b^2)$*, and default sets* $D(b, b^2)$*, and (ii) the price for short term bonds* $q^1(b', b^{2\prime}, x)$ *and long term bonds* $q^2(b', b^{2\prime}, x)$ *such that:*

1. *Taking as given the bond price functions* $q^1(b', b^{2\prime}, x)$ *and* $q^2(b', b^{2\prime}, x)$*, the policy functions* $b'(s)$*,* $b^{2\prime}(s)$ *and* $c(s)$*, repayment sets* $A(b, b^2)$*, and default sets* $D(b, b^2)$ *satisfy the representative domestic agent's optimization problem.*

2. *Bonds prices* $q^1(b', b^{2\prime}, x)$ *and* $q^2(b', b^{2\prime}, x)$ *are such that they reflect the domestic agent default probabilities and satisfy the lender's marginal rate of substitution.*

Term structure facts

Given that bond prices reflect the economy's default probabilities, the term structure of spreads in this model gives information on the timing of default.

1. In tranquil times long spreads are higher than short spreads.

If default events are forecasted for far in the future, the short rates spreads will be zero because tomorrow the likelihood of default is zero. However the spread on long bonds will be positive to compensate investors for a possible loss of principal in case of default when the bond is due. More formally assuming risk neutral lenders, $u_L(c_L) = c_L$, if the repayment set is the whole set, $A(b', b^{2\prime}) = X$, then annualized long rates are higher than short rates: $[q^2(b', b^{2\prime}, x)]^{1/2} \leq q^1(b', b^{2\prime}, x)$.

To see why this is, note that in this case $q^1(b', b^{2\prime}, x) = \delta$, and $q^2(b', b^{2\prime}, x) = \delta^2 \left[\int_X f(x', x) \int_{A(b'', b^{2\prime\prime}) \times B} Q(b'', b^{2\prime\prime}, s') f(x'', x') d(b'', b^{2\prime\prime}, x'') dx'\right] \leq \delta^2$ for $A(b'', b^{2\prime\prime}) \in X$.

We can think of this case as that of 'tranquil times' because default events are not foreseen in the near future. The prediction of the model is that in tranquil times, emerging economies would face higher long spreads than short spreads which is consistent with the data.

2. In crisis times short spreads are higher than long spreads.

If default events are forecasted for the next period, the short spread can be higher than the long spread if conditional on repaying tomorrow default events are avoided in the future. Even though default events next period also encompass default on long term debt, annualized yields on long bonds are smaller because in present value terms default events far in the future are less costly for lenders given that $\delta < 1$.

If the repayment set is less than the whole set, $A(b', b^{2\prime}) \in X$ and conditional on repaying tomorrow future repayment sets are the whole set , $A(b'', b^{2\prime\prime}) = X$ then short rates are higher than long rates: $[q^2(b', b^{2\prime}, x)]^{1/2} \geq q^1(b', b^{2\prime}, x)$. To illustrate this case note that when $u_L(c_L) = c_L$, $q^1(b', b^{2\prime}, x) = \delta \int_{A(b', b^{2\prime})} f(x', x) dx' \leq \delta$ and $q^2(b', b^{2\prime}, x) = \delta^2 \left[\int_{A(b', b^{2\prime})} f(x', x) dx'\right]$. Given that $\left[\int_{A(b', b^{2\prime})} f(x', x) dx'\right] \leq 1$ because $A(b', b^{2\prime}) \in X$, the annualized long yield is smaller than the short yield: $[q^2(b', b^{2\prime}, x)]^{1/2} \geq q^1(b', b^{2\prime}, x)$.

Role of long maturity debt on borrowing

In a standard incomplete markets model with fluctuating output and without default, a borrower might find the portfolio of long and short assets indeterminate if the risk free rate is constant across time. This is because the two assets are perfectly interchangeable given that their price and payoff structure is exactly the same. But if the risk free rate is time varying, as in the case of risk averse lenders, the borrower may have definite patterns of debt holdings for short and long maturities. For example if the short rate today is low, the borrower might have incentive to borrow more long term to lock in that low short rate and insure against future possible increases in the short rate. Thus this model encompasses this mechanism in the case of risk averse lenders.

However, in this default model even with constant risk free rate the borrower has incentives to hold a precise portfolio of both assets. Both assets are distinct because the effective returns for long and short bonds are different given the timing of default events. Also both assets give the borrower different hedging strategies because of future changes in prices after negative default news. In fact this price effect gives the borrower incentives to borrow relatively more early on when long bonds are available because this relaxes borrowing constraints in future low output times. To formalize how the introduction of long term bonds affects borrowing incentives let's analyze the following example.

Consider equilibrium consumption allocations when only short defaultable bonds are available and lenders are risk neutral. In particular let's consider the allocations on three consecutive nodes after histories: x^{t-1}, x^t, x^{t+1}.

Let's assume that in the third node for some particular realization of the shock after some history: $x_{j,t+1}|x^{t-1}, x_{j,t}$ default is chosen. Assume that for all other shock realizations and histories $x_{i,t+1}|x^{t-1}, x_{i,t}$ for all $i \neq j$, repayment is optimal . Also assume that for all histories x^{t-1} and x^t the borrower repays its debt and has access to financial markets.

Given our assumptions, equilibrium consumption for the case with only short bonds on these three nodes are:

$$c(x^{t-1}) = y(x^{t-1}) - b(x^{t-2}) + q(x^{t-1})b(x^{t-1})$$
$$c(x^t) = y(x^t) - b(x^{t-1}) + q(x^t)b(x^t)$$
$$c(x_{j,t+1}|x^{t-1}, x_{j,t}) = y^{def}(x^{t+2})$$
$$c(x^{t+1}) = y(x^{t+1}) - b(x^t) + q(x^{t+1})b(x^{t+1}) \quad \text{for all other } x^{t+1}$$

Now let's look at the effect of a variation where the consumption time path changes due to the introduction of long bonds in the first node only. Equilibrium consumptions for this variation in the three nodes are:

$$\widetilde{c}(x^{t-1}) = y(x^{t-1}) - b(x^{t-2}) + \widetilde{q}(x^{t-1})\widetilde{b}(x^{t-1}) + q^2(x^{t-1})b^2(x^{t-1})$$
$$\widetilde{c}(x^t) = y(x^t) - \widetilde{b}(x^{t-1}) + \widetilde{q}(x^t)\widetilde{b}(x^t)$$
$$\widetilde{c}(x^{t+1}) = y(x^{t+1}) - \widetilde{b}(x^t) - b^2(x^{t-1}) + q(x^{t+1})b(x^{t+1})$$

Let's now modify the short term positions, such that we keep all the consumption allocations exactly the same for all histories, except at node x^{t-1} and $x_{j,t}|x^{t-1}$.

Given that $\widetilde{c}(x^{t+1}) = c(x^{t+1})$ and that feasible debt positions are the same for all histories after x^{t+1}, optimal default choices are the same for all histories after x^{t+1}. Also our variation implies that $b(x^t) = \widetilde{b}(x^t) + b^2(x^{t-1})$ because $\widetilde{c}(x^{t+1}) = c(x^{t+1})$ and all future consumptions are equal. Also given that $\widetilde{c}(x_{i,t}|x^{t-1}) = c(x_{i,t}|x^{t-1})$ for all $i \neq j$ and that default is not optimal for all $x_{j,t+1}|x^{t-1}, x_{i,t}$, in the variation we get that $-\widetilde{b}(x^{t-1}) + \delta\widetilde{b}(x_{i,t}|x^{t-1}) = -b(x^{t-1}) + \delta b(x_{i,t}|x^{t-1})$.

Thus our modified consumptions in this variation at the two consecutive nodes, $\widetilde{c}(x^{t-1})$ and $\widetilde{c}(x_{j,t}|x^{t-1})$, can be written as the following:

$$\widetilde{c}(x_{j,t}|x^{t-1}) = y(x_{j,t}|x^{t-1}) - b(x^{t-1}) + \widetilde{q}(x_{j,t}|x^{t-1})b(x^t) + [\delta - \widetilde{q}(x_{j,t}|x^{t-1})]b^2(x^{t-1})$$

$$\widetilde{c}(x^{t-1}) = y(x^{t-1}) - b(x^{t-2}) + \delta b(x^{t-1}) + [q^2(x^{t-1}) - \delta^2]b^2(x^{t-1})$$

Default choices are the same so $\widetilde{q}(x_{j,t}|x^{t-1}) = q(x_{j,t}|x^{t-1})$ and the modified consumptions under this variation are equal to the original consumption plus an additional term that takes into account the long term debt:

$$\widetilde{c}(x_{j,t}|x^{t-1}) = c(x_{j,t}|x^{t-1}) + [\delta - q(x_{j,t}|x^{t-1})]b^2(x^{t-1})$$

$$\widetilde{c}(x^{t-1}) = c(x^{t-1}) - [\delta^2 - q^2(x^{t-1})]b^2(x^{t-1})$$

Note that the modified consumption will be different than the original consumption if bond prices change from one period to the next. In particular if the borrower moves to the node with positive default probabilities, the consumption in this node will be larger due to a positive effect of the reduced price in outstanding long debt. However this greater consumption in this period comes at a cost in terms of the previous period consumption. What happens is that in this first period the borrower effectively has to save short term a bit more than in the original consumption time path, and this extra savings are costly.

Now equilibrium prices given the default time path are the following

$$q(x_{j,t}|x^{t-1}) = \delta\left(1 - \pi(x_{j,t+1}|x^{t-1}, x_{j,t})\right)$$

$$q^2(x^{t-1}) = \delta^2\left[1 - \pi(x_{j,t}|x^{t-1})\pi(x_{j,t+1}|x^{t-1}, x_{j,t})\right]$$

where $\pi(x_{j,t+1}|x^{t-1}, x_{j,t})$ is the conditional probability of state $x_{j,t+1}$ given history $x^{t-1}, x_{j,t}$.

The net effect on lifetime utility from holding long term debt at history x^{t-1} is then given by:

$$\tfrac{dv^o}{db^2} = -\delta^2 u'(c(x^{t-1}))\pi(x_{j,t}|x^{t-1})\pi(x_{j,t+1}|x^{t-1}, x_{j,t}) + \beta\delta u'(c(x_{j,t}|x^{t-1}))\pi(x_{j,t+1}|x^{t-1}, x_{j,t})$$

$$\frac{dv^o}{db^2} = \delta^2\pi(x_{j,t+1}|x^{t-1}, x_{j,t})[\frac{\beta}{\delta}u'(c(x_{j,t}|x^{t-1})) - u'(c(x^{t-1}))\pi(x_{j,t}|x^{t-1})]$$

Thus holding long term debt can be beneficial due to the positive price effect if marginal utility in the pre-default period is high enough. For example, if in the pre-default period the borrower is at the borrowing constraint because of extremely high interest rates and low shocks, long term bonds can alleviate the constraints to some degree. Thus we expect our agent to borrow long term quite a bit in normal times (history x^{t-1} in this example) to relax constraints due to positive price effects on outstanding debt in future periods that feature positive default probabilities.

3.4 Quantitative Analysis and Data

3.4.1 Data

The first column of Table 3.4 shows business cycle statistics for the Brazilian economy. The series are quarterly for 1990-2004 deflated by CPI and taken from IBGE (Instituto Brasileiro de Geografia e Estatistica). The spread series for the long and short bond are the 5 year and 2 year spreads from the bond data discussed in section 2.[4] In Brazil consumption is as volatile as output, and short spreads are more volatile than output. Spreads for both short and long term bonds are negatively correlated with output and weakly positively correlated with the trade balance.

3.4.2 Parameter Values

The model is solved numerically to evaluate its quantitative predictions regarding the term structure of sovereign bonds in emerging markets and optimal maturity composition. In the benchmark model we assume that lenders are risk neutral and the parameters are calibrated to match certain features of the Brazilian economy.

The utility function of the borrower used in the numerical simulations is $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$. The risk aversion coefficient is set to 2 which is a common value used in real business cycle studies. The probability of reentering financial markets after default θ is set to 0.125

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[4] The statistics are not exactly equal to those of Table 3.2 because these are quarterly series to make them consistent with the business cycle statistics.

following Argentina's recent default experience where it took 2 years before this country re-enter international financial markets. This is consistent with the estimates of Gelos et al. (2002) who find that during the default episodes of the 1990s, economies were excluded from the credit markets only for a short period of time.

Output after the default before re-entering to financial markets is assumed to remain low and below some threshold. We assume output after default evolves in the following form:

$$h(y) = \left\{ \begin{array}{ll} y & if\ y \leq (1-\lambda)\overline{y} \\ (1-\lambda)\overline{y} & if\ y > (1-\lambda)\overline{y} \end{array} \right\}$$

The assumption that default entails output contractions and these are larger in good shocks can be rationalize by the fact that government default affects private foreign borrowing financing and this is disproportionately more costly in good productivity shocks. After a default from the government, investors might fear higher risks of expropriation, less domestic enforcement of contracts, high devaluations, etc., which would reduce private capital to finance projects in emerging countries. This would make output lower after default and importantly less responsive to productivity fluctuations (Tirole 2003, Cole and Kehoe 1998). The fact that private foreign capital decreases after sovereign defaults in consistent with the data in emerging countries where foreign private debt and equity decrease dramatically. We choose the output threshold λ to be equal to 0.02 and will perform sensitivity on this parameter.

The time preference parameter β is calibrated across the experiments such that the default probability in the limiting distribution is 3%. The stochastic process for output and the lenders consumption are assumed to be jointly distributed log-normal as AR(1) processes $\log(y_t) = \rho \log(y_{t-1}) + \varepsilon_t$, $\log(c_{L,t}) = \log(c_{L,t-1}) + \varepsilon_{L,t}$with $E[\varepsilon^2] = \eta_y^2$, $E[\varepsilon_L^2] = \eta_c^2$ and $E[\varepsilon' \varepsilon_L] = \eta_{cy}$. Shocks are calibrated to Brazil GDP. The lender's consumption growth rate is assumed to be i.i.d. in order to have constant risk free short rate. Shocks are discretized into a 18 state Markov chain by using a quadrature based procedure (Hussey and Tauchen 1991).

For the case of risk averse lenders, the utility function we use is $u_L(c_L) = \frac{c_L^{1-\sigma_L}}{1-\sigma_L}$ with $\sigma_L = 5$. Table 3.3 summarizes the parameter values.

3.4.3 Simulation Results

The model features several features of the term structure properties of foreign bonds in Brazil. Figure 3.4 presents the time series dynamics of the benchmark model prior to a default episode. Output and consumption are log and detrended series, and debt holdings and the trade balance are reported as a fraction of mean output. In period 21, the borrower chooses to default because of the low output shock.

The upper left panel shows dynamics of the annualized spreads on short and long bonds. The spreads on short bonds reflect immediate default probabilities. When default probabilities in the near future are low, the long spread is larger than the short spread because only the long spread forecasts future default events. However when default probabilities in the next period are high, the spread curve inverts with the short spread being larger than the long spread. The intuition for this result, as presented in the subsection on the model's term structure facts above, is that defaults on long term debt are less costly for lenders because they are due further in future. As the figure shows the model is able to mimic the dynamics of the spread curve in the data in that in tranquil times it is upward sloping and in crises it is inverted.

The upper right panel shows the dynamics of consumption and output. Both series are highly correlated but consumption is more volatile than output. The fact that consumption is more volatile than output in this model is not a feature of the multiple asset structure but is due to the default option and the incomplete markets. Given that default incentives are higher in recessions, with persistent shocks these are the times when interest rates are very high and the borrower is constrained. Thus in recessions very little borrowing is sustainable. However in booms interest rates are lower and given that the borrower discounts the future more than the lender, borrowing is optimal in booms.

The lower left panel shows the dynamics of short and long borrowing as a fraction of mean output. Prior to the default in periods 17 and 18, even though the short rate is lower than the long rate along the equilibrium, the economy borrows more long term. The reason is because long term borrowing is beneficial for completing markets and thus even if spreads are higher the borrower chooses to borrow more long.

The lower right panel presents the dynamics of the trade balance as a fraction of mean output. The trade balance is countercyclical and in periods prior to the default it is positive even though the economy is in a recession. The reason is that interest rates are too high and even though the borrower would like to borrow more it cannot.

The second column of Table 3.4 presents the business cycle statistics for the benchmark model. The statistics are taken from the limiting distribution of assets conditional on not defaulting and the series are treated equally as the data. The mean net foreign debt position is 7.3% of GDP.

The business cycles statistics confirm the above dynamics. Both spreads short and long are volatile in the model and the magnitudes match the data. The model also matches the relative volatility of spreads. Short spreads are twice as volatile as long spreads because on quite often they are lower but in crises they are higher than long rates. However the model predicts that on average short rates are equal to long rates. The reason is that with risk neutral pricing, the expectation hypothesis hold by construction which translate into an average flat spread curve.

The model matches the negative correlation of both spreads with output because default is more likely in recessions. With persistent shocks a low shock today predicts a low shock tomorrow and thus the borrower faces in this period higher interest rates. The model matches the positive correlation between spreads and the trade balance. The reason is that prior to default episodes the model produces large short rates and trade balance surpluses because the borrower is constrained. However in the data the correlation between spreads and the trade balance is much weaker than in the model.

The model generates a negative correlation between the trade balance and output. This feature is similar to that in Aguiar and Gopinath (2004) where the economy borrows in booms because of the expectation of higher future growth rates. Here what drives the result are the state contingent borrowing constraints that are tighter in recessions and the impatience effects. In recessions the economy would like to borrow, but in equilibrium in cannot because of the high yields and state contingent constraints being tight. Thus borrowing is small in recessions. In booms the economy wants to borrow when wealth is not too low because of the impatience effects. Given that borrowing constraints are state contingent, if the economy is exiting a recession the asset position is relatively high (because the borrowing constraints are tight), thus in booms the economy tends to borrow given the higher initial wealth.

Even though the economy borrows more in booms both short and long term because of the state contingent borrowing constraints, the relation is more pronounced with long term borrowing specially when interest rates are low. In particular when both interest rates are equal to the risk free rate, the economy borrows in booms only long term and on average saves in booms. The correlation between the trade balance and output in periods when both interest rates equal the risk free rate is 0.81 but the correlation between long term borrowing $b^{2\prime}$ and output in these periods is equal to 0.45. This is because long term borrowing also serves for relaxing constraints in future periods even if in the current period constraints are not tight (as the example in the previous section showed). The model then predicts than when interest rates are low the economy borrows mostly long term in booms.

This is consistent with the data of emerging markets.

Regarding debt issuances, we find that in the model short bonds are issued primarily in times of high short spreads (i.e. crises) and long bonds are used more primarily in periods of low short spreads. Short issuances are larger when spreads are above the mean level and long issuances are larger when spreads are below the mean level. On average the mean level of short issuances for high interest rate periods is 14.2% higher than average whereas for these periods long issuances are 3% below their mean level. Moreover when spreads are low the level of short issuances is 21% lower than its overall mean level, whereas long bonds issuances are 4.2% higher than its mean level. So in high spreads periods short bonds are used more aggressively and in low spread periods long bonds are used relatively more aggressively. Thus our model matches the patterns for bond issuances found in Brazil.

The feature that the benchmark model misses is the level of the short and long spreads. The average short and long spread in the model are 3.25% and 3.23% respectively which is lower than in the data where they are 9.93% and 12.18%. The model predicts that the spread level on both bonds is similar to the average default probability of the model given that in the benchmark lenders are risk neutral. In addition even though in the time series the model features the dynamics of the spread curve in tranquil times and crisis as in the data, it misses the relatively higher average spread on long bonds. This is because in the benchmark model both the risk free rate and average spread are similar for both maturities.

In the data, spread levels are much larger than default probabilities for most emerging markets. In fact, studies from corporate defaultable bonds find a similar disconnect. Huang and Huang (2003) document that in calibrated structural default models, default probabilities account for little of the spreads in corporate junk bonds. Thus a challenge for a model of sovereign defaultable bonds is getting simultaneously relatively low default probabilities together with high spreads. Candidates for mechanisms that have been identified to give rise to such high spreads in corporate defaultable bonds other than losses from default are:

risk premia, liquidity issues, term premia, and differential taxes and fees for investors. An empirical question in the sovereign bonds markets is identifying from the data how much of the spread should be accounted by each one of these components. We want to pursue this issue further, but as a first step we consider the role of risk aversion within the context of our model.

The third column in Table 3.4 shows statistics for the case of risk averse lenders. In our model pricing defaultable bonds under risk averse lenders increase significantly the level of spreads. For a calibrated 3% default probability, average spreads on long bonds are 11.26% and on short bonds are 11.80%. Thus risk aversion helps to break the link between default probabilities and spreads. The reason we get a considerably higher spread is the positive correlation assumed between the innovation between Brazilian output and the innovation of the lender's consumption growth rate. In this model defaults occur when the borrower faces a recession, and these are associated with states of higher marginal rate of substitution for the lender. Thus risk averse pricing compensates beyond the risk neutral default probability because default co-vary adversely with the pricing kernel. In the background the positive correlation between the lender's consumption growth and Brazilian output is thought of as direct wealth effects that a specialized investor would have when its portfolio is tied to Brazilian GDP. Pricing under risk averse lenders does not affect much the other business cycle statistics as the table shows.

However the risk averse specification misses the average spread curve observed in Brazil, delivering a flat average spread curve. The reason why risk aversion misses the average spread curve is due to the i.i.d. assumption on the lender's kernel. Although on average every period risk averse pricing delivers higher spreads, the relatively higher spread is the equal across all periods because the pricing kernel is i.i.d. Thus a challenge for our model is to have simultaneously constant (or very stable) risk free rate as in the data with time varying risk premia. We are currently exploring more fully this set up.

Another issue of interest is how default incentives change with the introduction of longer maturity debt. In particular, will default episodes be less likely and thus yields lower when long bonds are available to the sovereign borrower? The answer to this question under the light of our model is no. Default probabilities and spreads in a model with only one period bonds and equal parameters as our benchmark model are lower. The default probability in such model is equal to 1.9% and the average spread of the short bond is 1.98%.

The reason why long term borrowing does not reduce default episodes is that default premia in our model has only to do with the borrower's side (at least with risk neutral pricing as in the benchmark). Thus the model abstracts from external factors and shocks for which long term borrowing can provide the benefits of managing external sudden stops. The reason why long term borrowing increases the likelihood of default events is more subtle and has to do with how borrowing incentives change. Long bonds provide extra benefits from borrowing because of future changes in bond prices. If the agent borrows long term today, tomorrow that two-period bond is equivalent to a one-period bond. Thus if tomorrow default probabilities become positive the effective cost is lower because of a lower price on one period bonds. The agent is then more likely to engage in risky borrowing specially if the consumption in the pre default period is low. Of course welfare increases with the introduction of long term bonds, but default premia does not decrease precisely because of the extra benefits of borrowing long term.

3.5 Conclusion

This paper has constructed a dynamic model of borrowing and default to study the term structure of sovereign bond spreads. In the data, these spreads are volatile, and spreads on long term bonds are on average higher than on short-term bonds. This pattern inverts during a crisis. In our model, spreads on long-term bonds are higher during tranquil times because the only risk of default occurring is far into the future. In a crisis, the risk of default is currently high, raising the short-term spread, but if the economy avoids default, then it becomes much more likely to repay its debt, and the long-term spread reflects a relatively lower risk of default. Because of the benefits of issuing long-term bonds in the presence of default risk, the model also generates the pattern of bond issuances observed in the data, that short-term debt is used more heavily in a crisis.

3.6 Appendix: Computing Brazilian Spreads

This appendix describes the calculation of spread curves for the Brazilian government bonds mentioned in section 2 of the paper. First, for each bond, the annualized yield-to-maturity is computed at each date a price is quoted. The yield y_t^n at date t on a coupon bond with n coupon periods left to maturity, given the price p_t^n solves:

$$p_t^n = \sum_{j=0}^{n-1} \frac{c_j}{\left(1+\frac{y_t^n}{F}\right)^{w_t+j}} + \frac{100}{\left(1+\frac{y_t^n}{F}\right)^{w_t+n-1}} - c_j\left(1-w_t\right)$$

In the formula above, w_t is the fraction of a coupon period until the next coupon payment, F is the frequency of coupons (1 for annual coupons and 2 for semiannual coupons) and c_j is the coupon payment at each future coupon date j. The first term is the present value of coupon payments discounted by the yield (including accrued interest when the settle date t is between coupons). The second term is the present value of the principal payment at maturity.[5] The third term subtracts accrued interest.

The spread s_t^n is calculated as the difference between the yield and the yield of a corresponding risk-free bond[6],

$$s_t^n = y_t^n - \bar{y}_t^n$$

The risk-free yields are obtained from time series of constant-maturity yields. However, since, for any time period t, the time-to-maturity n of the sovereign bonds is generally not an even number of years, the risk-free yield over which to form the spread is taken from an interpolation of the even constant-maturity risk-free yields, following Nelson and Siegel

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[5] The yield discounting the principal may be different from the yield discounting coupon payments if there are guarantees on the principal. Some bonds, for example, are collateralized by US treasury notes of the same maturity. Then, the yield used to discount the principal is the US treasury yield of maturity n at time t, denoted $\bar{y}_t^n$. In this case, y_t^n is referred to as the *stripped* yield.

[6] For sovereign bonds denominated in dollars, the yield $\bar{y}_t^n$ used is that of a US treasury note of maturity n at time t. For bonds denominated in Euros, Deutschemarks, French Francs, Austrian Schillings, Dutch Guilders, British pounds or Italian Lira, the yield $\bar{y}_t^n$ used is that of a European central bank note of maturity n at time t.

(1987). This procedure obtains yields as a smooth function of maturity by regressing the even-maturity yields at each date on functions of the time to maturity:

$$\bar{y}_t^n = \bar{\beta}_{1t} + \bar{\beta}_{2t}\left(\frac{1-e^{-\lambda n}}{\lambda n}\right) + \bar{\beta}_{3t}\left(\frac{1-e^{-\lambda n}}{\lambda n} - e^{-\lambda n}\right)$$

We fix the parameter λ to be 0.06, as in Diebold and Li (2006), and $\bar{\beta}_{1t}$, $\bar{\beta}_{2t}$, and $\bar{\beta}_{3t}$ are estimated by OLS for each period t.

Once spreads are calculated for individual bonds, a spread curve over maturities is interpolated for each date in the same way as the risk-free yield curve, estimating the following equation:

$$s_t^n = \beta_{1t} + \beta_{2t}\left(\frac{1-e^{-\lambda n}}{\lambda n}\right) + \beta_{3t}\left(\frac{1-e^{-\lambda n}}{\lambda n} - e^{-\lambda n}\right)$$

To ensure variation in the maturities available at each date, only certain dates are used: those for which both short-term (less than 2 years to maturity) and long term (more than 10 years to maturity) prices are available, and for which the total number of bond prices available is at least 8. This leaves us with a date range of November 29, 1996 to March 24, 2006, with two short gaps in late 1998.

Table 3.1: Brazil Bond Issuances

Date	Bonds Issued	Amount (million USD)	Average Maturity (years)
Apr 1994	15	74517	20.8
Nov 1996-Oct 1997	14	8013	18.6
Nov 1997-Nov 1999	9	9604	6.6

Table 3.2: Average Spread Term Structure

spreads	30 year	20 year	10 year	5 year	3 year	2 year
$\overline{s}$	12.68	12.55	12.17	11.4	10.37	9.10
σ_s	3.92	3.92	3.96	4.07	4.31	4.73

Table 3.3: Parameters

Discount factor lender	$\delta = 0.99$	U.S. quarterly interest rate 1%
Probability of re-entry	$\theta = 0.125$	Exclusion time 2 years
Output after default	$\lambda = 0.02$	
Risk aversion borrower	$\sigma = 2$	
Stochastic structure	$\rho = 0.9$, $\eta = 0.0235$	Brazil output
Risk neutral lenders		
Discount factor borrower	$\beta = 0.9435$	3% default probability
Risk averse lenders		
Stochastic structure	$\rho^L = 1$, $\eta^L = 0.014$	
	$\rho_{\eta^y \eta^L} = 0.30$	
Risk aversion lender	$\sigma_L = 5$	
Discount factor borrower	$\beta = 0.923$	3% default probability

Table 3.4: Business Cycles in the Data and Model Economies

	Brazil Data	Risk Neutral	Risk Averse
spr^s	9.93	3.25	11.80
spr^L	12.18	3.23	11.26
σ_{spr^s}	4.56	6.53	6.29
σ_{spr^L}	3.81	3.21	3.43
σ_y	5.38	5.30	5.2
σ_c	5.17	6.01	5.87
$\sigma_{(tb/y)}$	3.15	1.82	2.05
σ_{y,spr^s}	-0.18	-0.22	-0.07
σ_{y,spr^L}	-0.29	-0.31	0.01
$\sigma_{y,tb}$	-0.38	-0.25	-0.20
σ_{tb,spr^s}	0.07	0.37	0.36
σ_{tb,spr^L}	0.10	0.51	0.42
Default Prob		2.97	3.0

Figure 3.1: Spread curve in crisis and normal periods

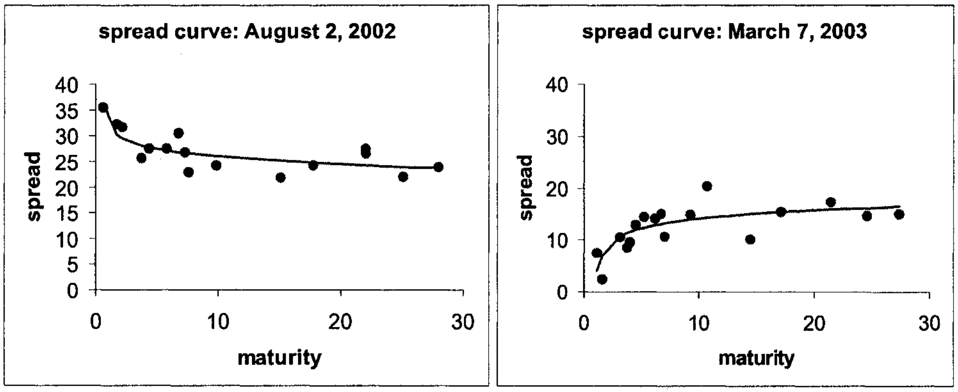


Figure 3.2: Time series of 2-year and 5-year spreads

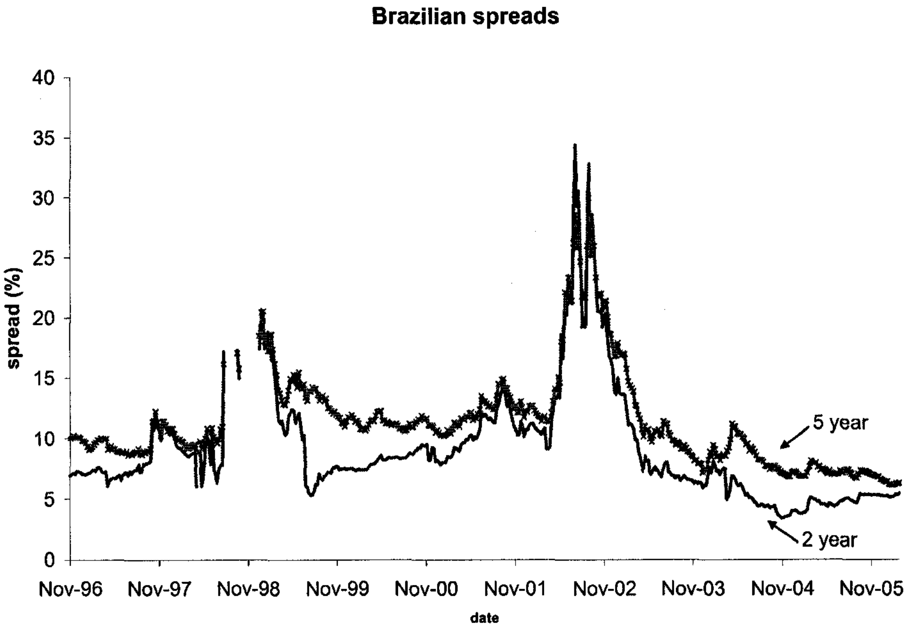


Figure 3.3: Issuances

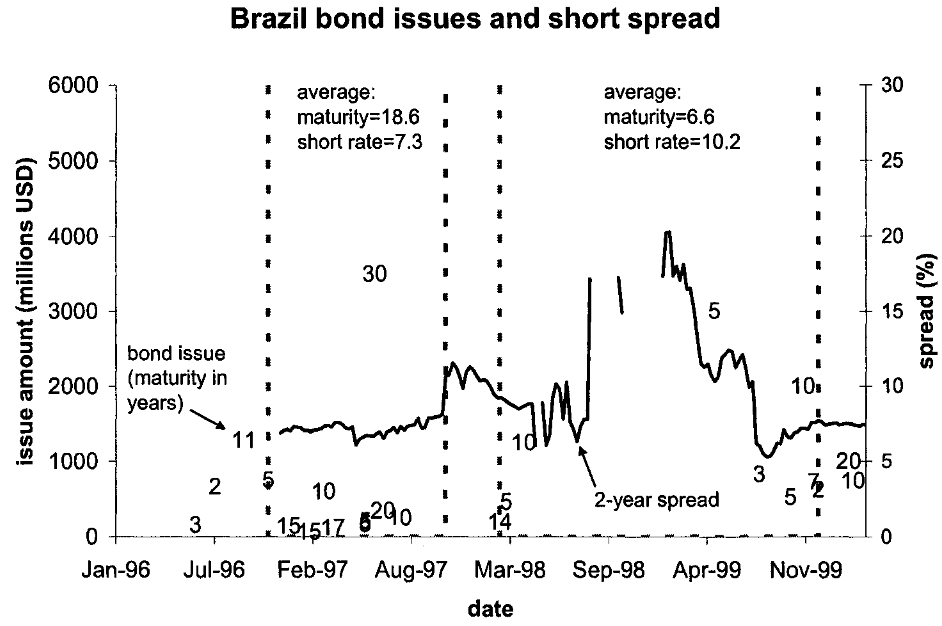
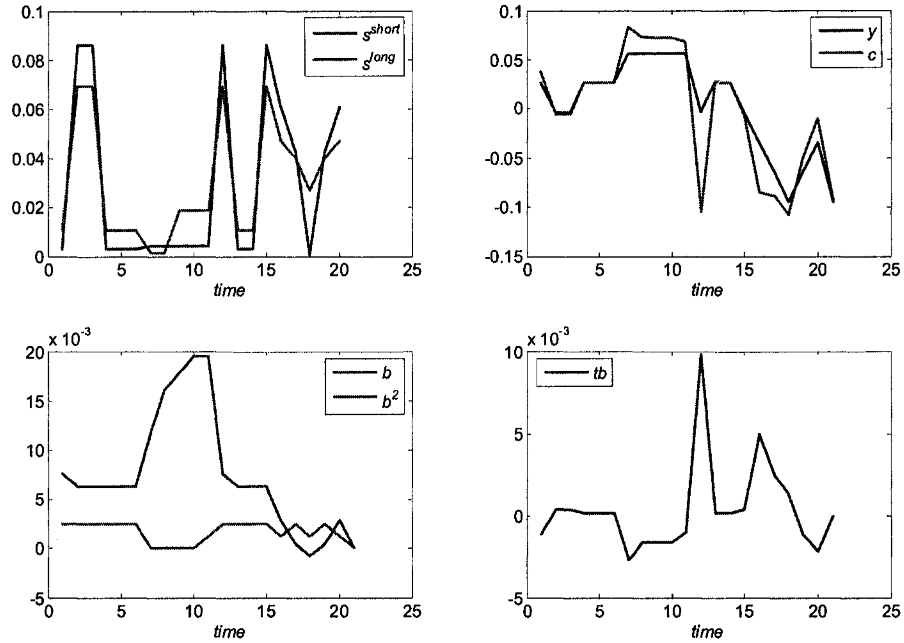


Figure 3.4: Time series dynamics from benchmark model



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